Influence of heterogeneous age-group contact patterns on critical vaccination rate for herd immunity

(joint work with C. Scoglio, KSU)

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Joan Saldaña

Universitat de Girona

Outline

1. Introduction: Social contact networks
2. The model and the basic reproduction number
3. Some results

Introduction

How much vaccine is required by any given country year by year to create herd immunity to block the transmission of a virus within a population?

If immunity is short lived $\rightarrow$ vaccination strategies: maximize the population coverage, minimize the number of deaths/cases/hospital burden, optimal containment of potential outbreaks, minimize the expected years of life lost due to deaths in each age groups, etc.

Vaccination strategy: define a prophylactic distribution of a limited stockpile of vaccine year by year.

Take into account the *population heterogeneity*.

Demography (2008)

Age group	Belgium	Germany	Italy	Netherlands	Peru	Zimbabwe
$[0, 18)$	0.207	0.179	0.171	0.219	0.361	0.491
$[18, 60)$	0.572	0.573	0.577	0.588	0.550	0.465
$60+$	0.221	0.249	0.251	0.193	0.089	0.044

Population fraction f_i for each age group in the considered countries.

The social contact matrix

c_{ij}: mean contact rate between a susceptible of age group i and individuals of age group j

$\hookrightarrow C = (c_{ij})$ is the social contact matrix

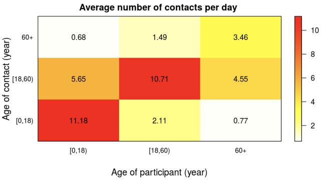
It is the central ingredient in our study.

β: transmission probability through an infectious contact (S-I).

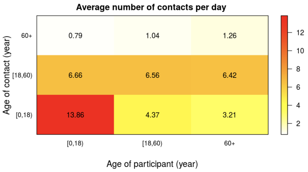
<u>Incidence term</u>:

$$\beta c_{ij} S_i \frac{I_j}{N_j}$$

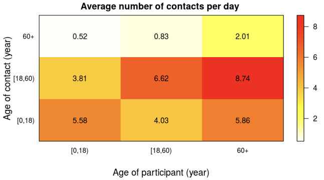
Examples of social contact matrices[1]



(a) The Netherlands



(b) Perú



(c) Zimbabwe

[1] http://www.socialcontactdata.org/socrates/

The SIRV model

Assuming an arbitrary vaccination strategy $\{w_i\}_{i=1}^{3}$, and ignoring the demographics, the equations governing the epidemics dynamics are

$$
\begin{aligned}
\frac{dS_i}{dt} &= -\sum_{j=1}^{3} \beta c_{ij} S_i \frac{I_j}{N_j} + \delta_i R_i + \delta_i^v V_i - p_i w_i \frac{S_i}{N_i}, \\
\frac{dI_i}{dt} &= -\sum_{j=1}^{3} \beta c_{ij} S_i \frac{I_j}{N_j} - \gamma_i I_i, \\
\frac{dR_i}{dt} &= \gamma_i I_i - \delta_i R_i - p_i w_i \frac{R_i}{N_i}, \\
\frac{dV_i}{dt} &= p_i \frac{w_i}{N_i} \left(S_i + R_i\right) - \delta_i^v V_i,
\end{aligned}
$$

with $S_i + I_i + R_i + V_i = N_i$, $i = 1, 2, 3$.

If $s_i = \frac{S_i}{N_i}$, $y_i = \frac{I_i}{N_i}$, $r_i = \frac{R_i}{N_i}$, $v_i = \frac{V_i}{N_i}$, and neglecting the last equation

$$\frac{ds_i}{dt} = -\sum_{j=1}^{3} \beta c_{ij} s_i y_j + \delta_i r_i + \delta_i^v v_i - p_i \bar{w}_i s_i, \tag{1}$$

$$\frac{dy_i}{dt} = -\sum_{j=1}^{3} \beta c_{ij} s_i y_j - \gamma_i y_i, \tag{2}$$

$$\frac{dr_i}{dt} = \gamma_i y_i - \delta_i r_i - p_i \bar{w}_i r_i, \tag{3}$$

with $s_i + y_i + r_i + v_i = 1$, and $\bar{w}_i := \frac{w_i}{N_i}$ $(i = 1, 2, 3)$ is the per capita vaccination rate of age group i.

From the constraint $\sum_{i=1}^{3} w_i = w$ and the definition of $\bar{w}_i$, it follows that $\sum_{i=1}^{3} \bar{w}_i f_i = \frac{w}{N} =: \bar{w}$, the mean per capita vaccination rate.

The Disease-free equilibrium

The disease-free equilibrium (DFE) of system (1)-(3) is $(s_i^*, 0, 0)$ with

$$s_i^* = \frac{\delta_i^v}{p_i \bar{w}_i + \delta_i^v}, \quad v_i^* = 1 - s_i^* \qquad (i = 1, 2, 3)$$

$\hookrightarrow$ Only susceptible and vaccinated individuals are present.

Maximum vaccination coverage

The condition on the per capita vaccination rates $\bar{w}_i$ for having a maximum vaccination coverage of the population at the DFE $\rightarrow$ minimize the susceptible population at the DFE: $\overline{s^*} = \sum_{i=1}^{3} f_i s_i^*$.

The condition $\nabla \overline{s^*}(\bar{w}_1, \bar{w}_2) = (0, 0)$ and the positivity of the rates amount to

$$\frac{p_1 \bar{w}_1 + \delta_1^v}{\sqrt{p_1 \delta_1^v}} = \frac{p_2 \bar{w}_2 + \delta_2^v}{\sqrt{p_2 \delta_2^v}} = \frac{p_3 \bar{w}_3 + \delta_3^v}{\sqrt{p_3 \delta_3^v}} \tag{4}$$

with the constraint $\sum_{i=1}^{3} \bar{w}_i f_i = \bar{w}$.

From these equations for two of the three $\bar{w}_i$, one easily obtains an explicit expression for the solution $\bar{w}_i^*$.

From (4) we have that, if the rate of immunity loss is the same for the vaccinated individuals of all the age groups and the probability of being protected after vaccination is the same across age groups, then the vaccination rates that guarantee the maximum fraction of vaccinated population are $\bar{w}_i = \bar{w}$.

$\hookrightarrow$ This corresponds to a *uniformly random mass vaccination*.

R_0 and R_0^*

The basic reproduction number for an age-structured epidemic model is given by the dominant eigenvalue of the so-called *next-generation matrix* N_g at the DFE without vaccination (R_0) and with it (R_0^*).

Assuming the same β for all the age groups and without vaccination,

$$N_g = \beta\, C\, \mathrm{diag}(1/\gamma_i)$$

where γ_i is the recovery rate of the age group i.

R_0 and R_0^*

To compare the impact of the vaccination strategy, in each country β is chosen to give $R_0 = 2.5$.

In the presence of vaccinated individuals, $s_i^* < 1$ and N_g becomes

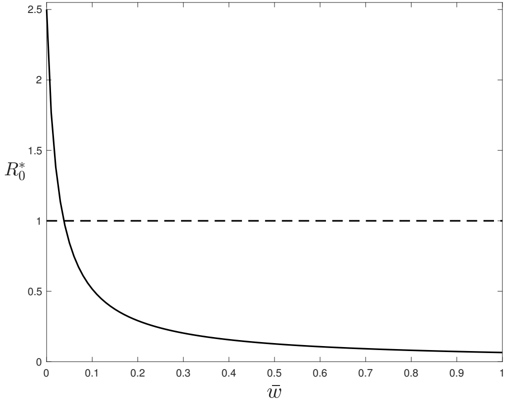
$$N_g^* = \beta\,\mathrm{diag}(s_i^*) C\,\mathrm{diag}(1/\gamma_i)$$

where s_i^* is given by the DFE of the model $\left(s_i^* + v_i^* = 1\right)$.

$\hookrightarrow$ R_0^* is the dominant eigenvalue of N_g^*: $R_0^*(\bar{w}_1, \bar{w}_2)$

R_0^* and uniform mass vaccinatiom

In this case, $\bar{w}_i = \bar{w}$ $(i = 1, 2, 3)$ and R_0^* only depends on $\bar{w}$:

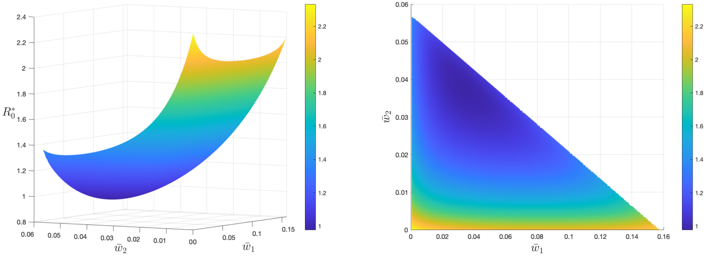


Examples of critical vaccination rates

Data set	$\bar{w}_c$	$V_{\bar{w}_c}$	$\bar{w}_c$	$V_{\bar{w}_c}$
Belgium	3.1222	59.17	3.2678	59.00
Germany	3.1487	59.38	3.2836	59.05
Italy	3.2765	60.37	3.3823	59.78
Netherlands	3.3374	60.87	3.4246	60.28
Peru	3.5408	62.03	3.5898	61.48
Zimbabwe	3.2882	59.74	3.4161	59.99

Table 1: Vaccination coverage (in %) adopting the uniformly random vaccination strategy at the critical per capita vaccination rate $\bar{w}_c$ (in %) with a 100% vaccine efficacy (left), and with $p_1 = 1$, $p_2 = 0.95$, $p_3 = 0.9$ (right). Parameters: $\gamma_1 = 1$, $\gamma_2 = 1$, $\gamma_3 = 0.9$, $\delta_1^v = 1/40$, $\delta_2^v = 1/52$, $\delta_3^v = 1/40$. For each country, β is scaled such that $R_0 = 2.5$ without vaccination.

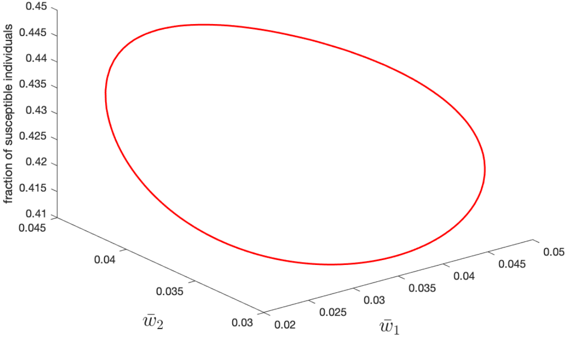
Example of a graph of $R_0^*(\bar{w}_1, \bar{w}_2)$



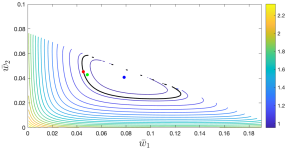
Belgium

Constraint on the vaccination rates: $\bar{w}_1 + \bar{w}_2 + \bar{w}_3 = \bar{w}_c$

$S(\bar{w}_1, \bar{w}_2)$ at $R_0^* = 1$



Contour plots of R_0^*



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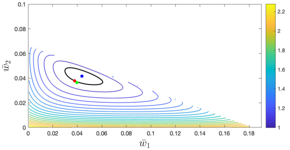
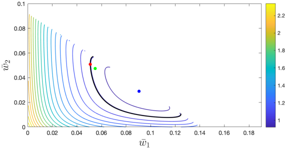


(b) Belgium



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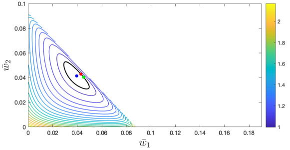

(d) Zimbabwe

Vaccination coverages

Data set	$R_0^{\min}$	$\bar{w}_1^0$	$\bar{w}_2^0$	$\bar{w}_3^0$	$V_{R_0^{\min}}$	$\bar{w}_1^*$	$\bar{w}_2^*$	$\bar{w}_3^*$	$V_{\max}$
Belgium	0.977	3.26	3.59	1.78	58.16	3.19	3.07	3.19	59.17
Germany	0.957	4.04	3.59	1.50	57.63	3.22	3.09	3.22	59.38
Italy	0.914	5.46	3.51	1.26	57.46	3.36	3.22	3.36	60.36
Netherlands	0.924	5.13	3.27	1.50	59.00	3.43	3.27	3.43	60.87
Peru	0.874	6.04	2.16	1.97	58.53	3.64	3.26	3.64	62.03
Zimbabwe	0.977	2.88	3.39	6.86	59.15	3.35	3.21	3.35	59.74

Table 2: Mean vaccination coverage (in %) at the vaccination strategy $\{\bar{w}_i^0\}$ (in %) leading to the minimum R_0 at the DFE, and at the vaccination strategy $\{\bar{w}_i^*\}$ (in %) computed from Eq. (5) leading to the maximum mean vaccination coverage. In both cases, the mean per capita vaccination rate $\bar{w} = \bar{w}_c$, and 100% vaccine efficacy is assumed. Parameters: $\gamma_1 = 1$, $\gamma_2 = 1$, $\gamma_3 = 0.9$, $\delta_1^v = 1/40$, $\delta_2^v = 1/52$, and $\delta_3^v = 1/40$. For each country, β is scaled such that $R_0 = 2.5$ in the absence of vaccinated individuals.