The Recoverability of Network Controllability

Anqi Chen
Group: NAS

Supervisor: Prof. Dr. Robert Kooij
Daily Supervisor: Peng Sun

Outline

- 1. Introduction
- 2. Purpose
- 3. Work
- 4. Conclusion

Introduction

1. Introduction

2. Purpose

3. Work

4. Conclusion

The Recoverability of Controllability

What is Controllability?

What is Controllability of Networks?

- Control Theory + Network Science
- Driver nodes

1. Kalman, R. E. (1963). Mathematical description of linear dynamical systems. Journal of the Society for Industrial and Applied Mathematics, Series A: Control, 1(2), 152-192.
2. Lombardi, A., & Hörnquist, M. (2007). Controllability analysis of networks. *Physical review. E, Statistical, nonlinear, and soft matter physics*, *75*(5 Pt 2), 056110.

TU Delft

The Recoverability of Controllability

1. Introduction

2. Purpose

3. Work

4. Conclusion

How?

Controllability of Networks → Maximum Matching problem

- ✓ Number of driver nodes(N_D) = Unmatched nodes

Controllability of Networks in Analytical Expression

- ✓ Fraction of driver nodes(n_D): $n_D = G_{out}(1-\hat{w}_1) + G_{in}(w_2) - 1 + k\hat{w}_1(1-w_2)$

$$k = \langle k_{in} \rangle = \langle k_{out} \rangle$$

$$G_{out}(x) = \sum_{k_{out}=0}^{\infty} P_{out}(k_{out}) x^{k_{out}}$$

$$G_{in}(x) = \sum_{k_{in}=0}^{\infty} P_{in}(k_{in}) x^{k_{in}}$$

$$w_2 = 1 - H_{out}(1-\hat{w}_1)$$

$$\hat{w}_1 = H_{in}(w_2)$$

$$H_{out}(x) = \sum_{k_{out}=1}^{\infty} \frac{k_{out} P_{out}(k_{out})}{< k_{out} >} x^{k_{out}-1}$$

$$H_{in}(x) = \sum_{k_{in}=1}^{\infty} \frac{k_{in} P_{in}(k_{in})}{< k_{in} >} x^{k_{in}-1}$$

Liu, Y. Y., Slotine, J. J., & Barabási, A. L. (2011). Controllability of complex networks. *Nature*, *473*(7346), 167–173.

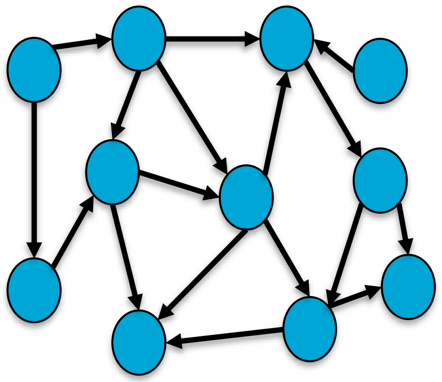
1. Introduction
2. Purpose
3. Work
4. Conclusion

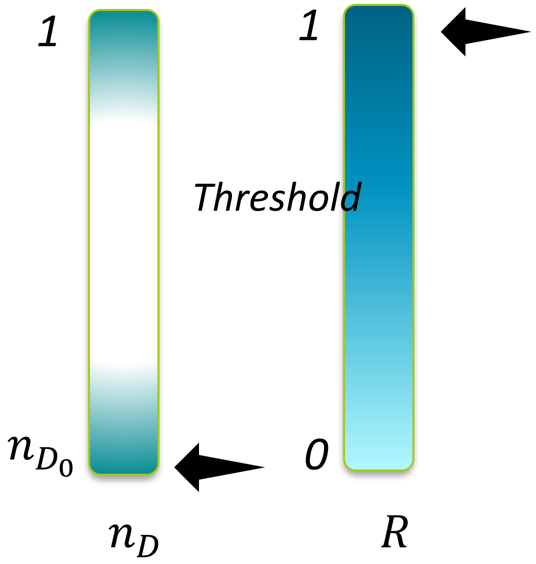
The Recoverability of Controllability

$$R = \frac{1 - n_D}{1 - n_{D_0}} \quad R \in [0,1]$$

n_{D_0}: fraction of driver nodes at the start

n_D: fraction of driver nodes during the attack\recovery process





- **Scenario A:**
  recovery of any alternative link
- **Scenario B:**
  recovery of attacked links

1. P. Van Mieghem, C. Doe, H.Wang, J.Martin Hernandez, D. Hutchison, M. Karaliopoulos and R. E. Kooij, 2010, "A Framework for Computing Topological Network Robustness", Delft University of Technology, report20101218.
2. He, Z., Sun, P., & Van Mieghem, P. (2019, October). Topological approach to measure network recoverability. In 2019 11th International Workshop on Resilient Networks Design and Modeling (RNDM) (pp. 1-7). IEEE.

Purpose

Purpose

- 1. **Analytically** express the controllability for the network *after removing/adding links*

- 2. **Analytically** approximate n_D and compute R-value *during attack process, and recovery process in Scenario A & B*

- *3. Compare different recovery strategies in Scenario A & B*

TU Delft

Work

1. Introduction

2. Purpose

3. Work

4. Conclusion

Networks used in work

Swarm Signalling Networks (SSNs):

- $P(k_{out}) = \delta(k - k_{out})$
- $P(k_{in}) = e^{-k} \frac{k^{k_{in}}}{k_{in}!}$
- $G_{out}(x) = x^{k}$
- $G_{in}(x) = e^{-k(1-x)}$

1. Komareji, M., & Bouffanais, R. (2013). Resilience and controllability of dynamic collective behaviors. PLoS one, 8(12), e82578.
2. http://www.topology-zoo.org/

SSN after removing links

1. Introduction

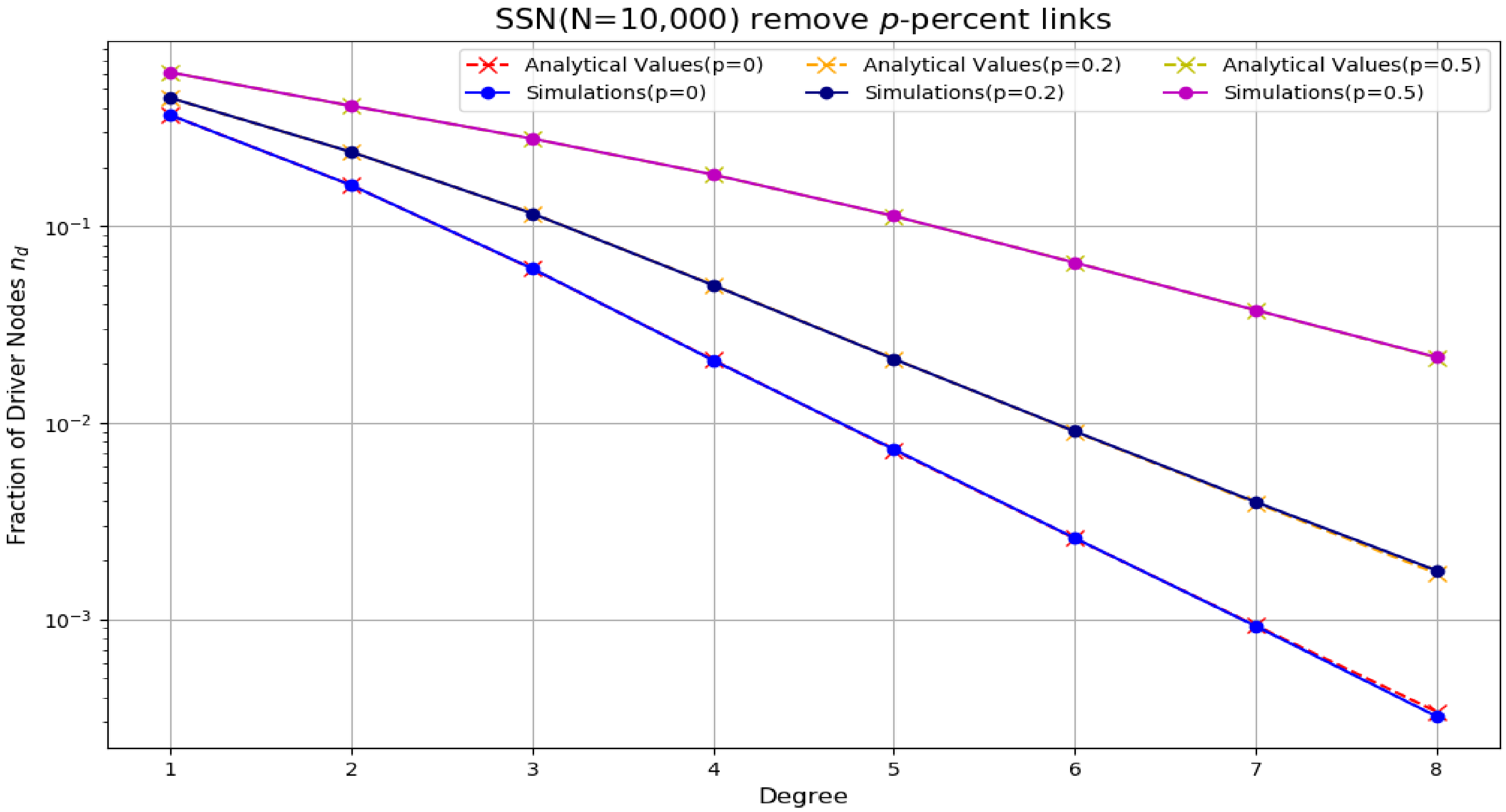
2. Purpose

3. Work

4. Conclusion

$$n_D = (p + (1-p)(1 - e^{-k(1-p)(1-w_2)}))^k - 1 + e^{-k(1-p)(1-w_2)} + k(1-p)(1-w_2)e^{-k(1-p)(1-w_2)}$$

$$p = \frac{m}{L}$$



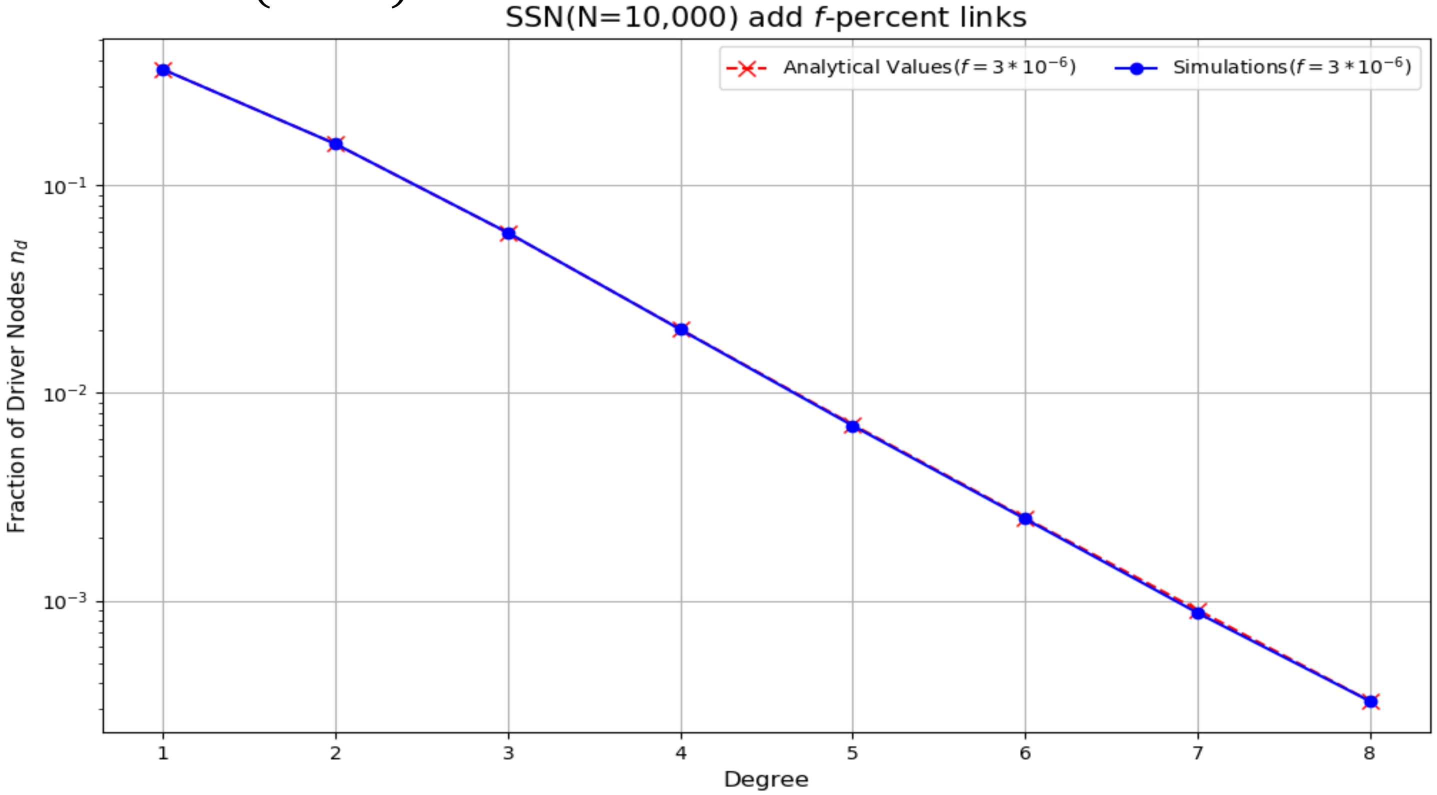
TU Delft

SSN after adding links

$$n_D = e^{-\bar{k}(1-w_2)} + (1 - e^{-\bar{k}(1-w_2)})^k (1 - f e^{-\bar{k}(1-w_2)})^{N-1-k} - 1 + \bar{k}(1-w_2) e^{-\bar{k}(1-w_2)}$$

$$\bar{k} = k + f(N-1-k)$$

$$f = \frac{m}{N(N-1) - L}$$



1. Introduction

2. Purpose

3. Work

4. Conclusion

TU Delft

Recoverability

1. Introduction

2. Purpose

3. Work

4. Conclusion

How to express the degree distributions after removal/recovery of links?

Scenario A: recovery of any alternative link

$$G(x)\begin{cases} attack: \bar{G}(x) = G(p + (1-p)x) \\ recovery: \bar{\bar{G}}(x) = (1 - f(1-x))^{N-1} * \bar{G}\left(\frac{x}{1 - f(1-x)}\right) \end{cases}$$

Scenario B: recovery of attacked links

$$G(x)\begin{cases} attack: \bar{G}(x) = G(p + (1-p)x), \quad p = \frac{i}{L}, 0 < i \leq M \\ recovery: \bar{\bar{G}}(x) = \bar{G}(p + (1-p)x), \quad p = \frac{2M - i}{L}, M < i \leq 2M \end{cases}$$

TU Delft

SSN's recoverability

Scenario A

Original Network

- $G_{out}(x) = x^k$
- $G_{in}(x) = e^{-k(1-x)}$
- $n_d = G_{in}(1-\omega_1) - 1 + G_{out}(\widehat{\omega_2}) + k \cdot \omega_1(1-\widehat{\omega_2})$

Attack Phase

- $G_{out}(x) = (p + (1-p)x)^k$
- $G_{in}(x) = e^{-k(1-p)(1-x)}$
- $n_d = G_{in}(1-\omega_1) - 1 + G_{out}(\widehat{\omega_2}) + k(1-p) \cdot \omega_1(1-\widehat{\omega_2})$

Recovery Phase

- $G_{out}(x) = (1-f(1-x))^{N-1}(p + (1-p)\frac{x}{1-f(1-x)})^k$
- $G_{in}(x) = (1-f(1-x))^{N-1} * e^{-k(1-p)(1-\frac{x}{1-f(1-x)})}$
- $n_d = G_{in}(1-\omega_1) - 1 + G_{out}(\widehat{\omega_2}) + (k(1-p) + f(N-1-k(1-p))) \cdot \omega_1(1-\widehat{\omega_2})$

TU Delft

SSN's recoverability

Scenario B

Original Network

- $G_{out}(x) = x^k$
- $G_{in}(x) = e^{-k(1-x)}$
- $n_d = G_{in}(1-\omega_1) - 1 + G_{out}(\widehat{\omega_2}) + k \cdot \omega_1(1-\widehat{\omega_2})$

Attack Phase

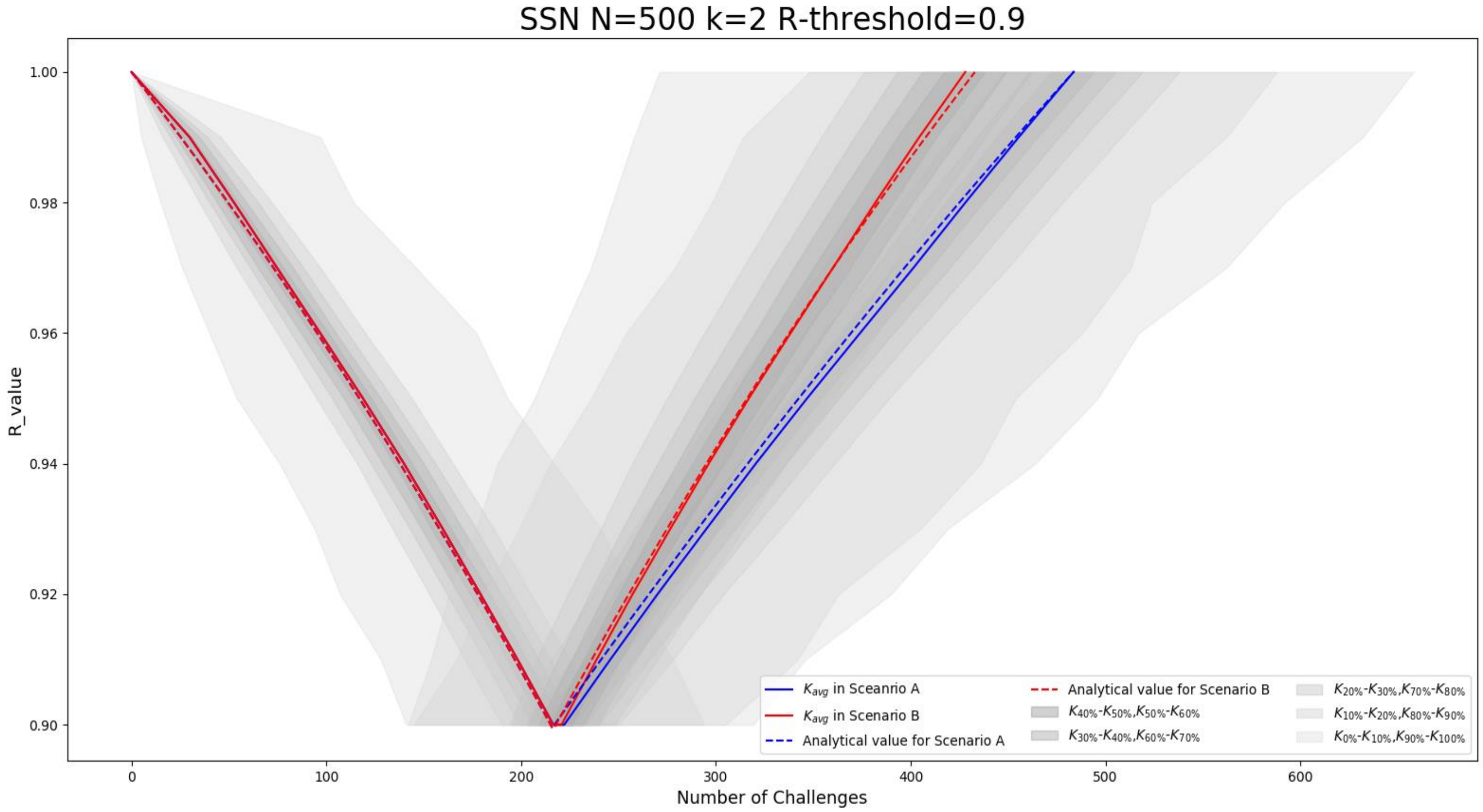
- $G_{out}(x) = (p + (1-p)x)^k$
- $G_{in}(x) = e^{-k(1-p)(1-x)}$
- $n_d = G_{in}(1-\omega_1) - 1 + G_{out}(\widehat{\omega_2}) + k(1-p) \cdot \omega_1(1-\widehat{\omega_2})$
- $p = \frac{i}{L}, 0 < i \leq M$

Recovery Phase

- $G_{out}(x) = (p + (1-p)x)^k$
- $G_{in}(x) = e^{-k(1-p)(1-x)}$
- $n_d = G_{in}(1-\omega_1) - 1 + G_{out}(\widehat{\omega_2}) + k(1-p) \cdot \omega_1(1-\widehat{\omega_2})$
- $p = \frac{2M-i}{L}, M < i \leq 2M$

SSN's Recoverability

1. Introduction
2. Purpose
3. Work
4. Conclusion



TU Delft

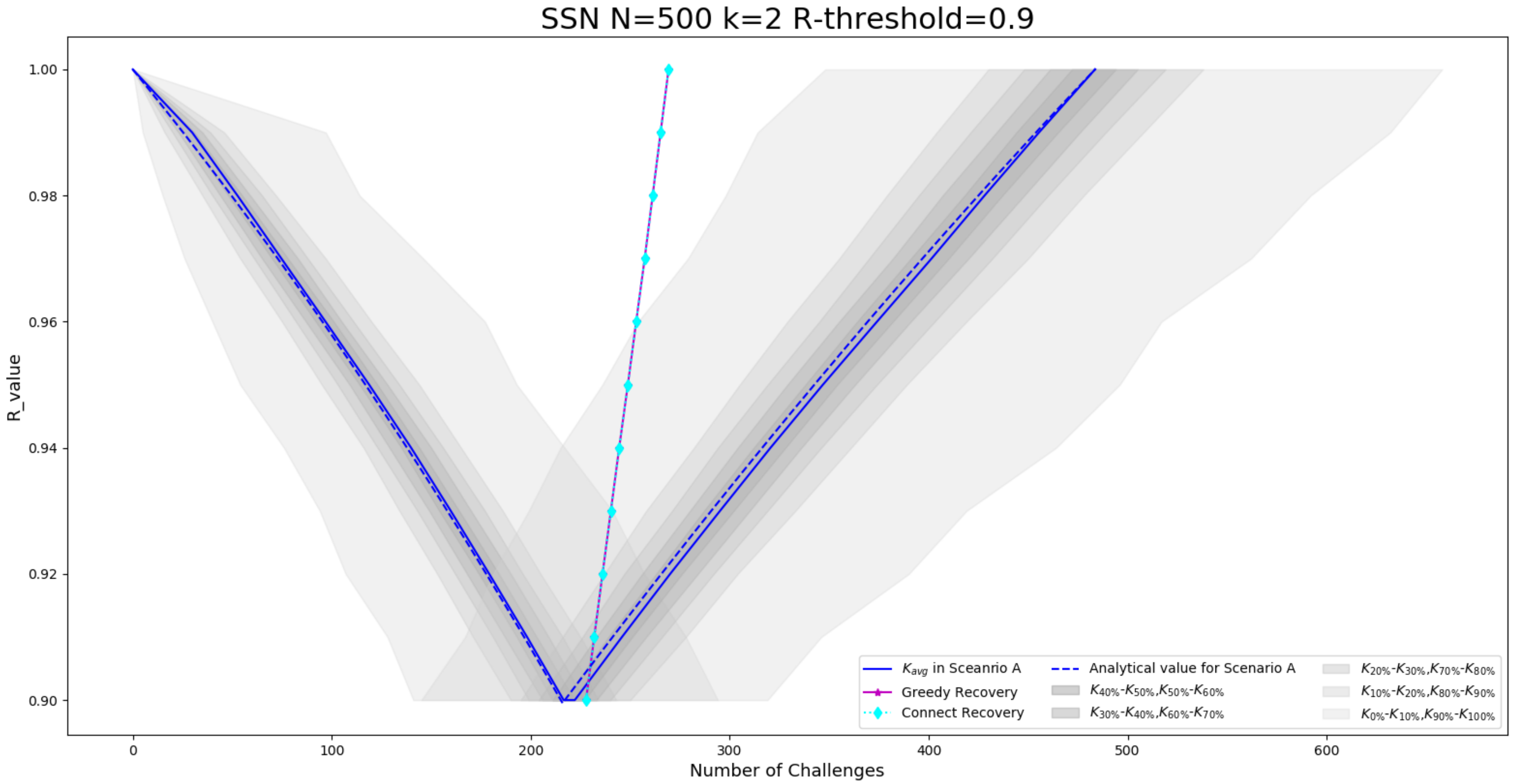
Recover Strategies-scenario A

1. Introduction

2. Purpose

3. Work

4. Conclusion



Random: 15.2 s/time
Greedy: 8531.9 s/time
Connect: 0.04 s/time

TU Delft

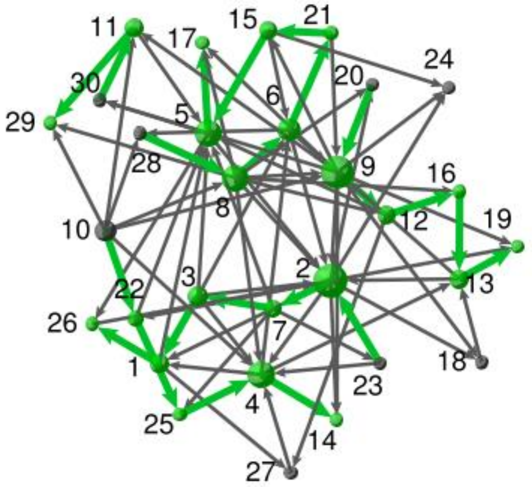
Connect Recovery

1. Introduction

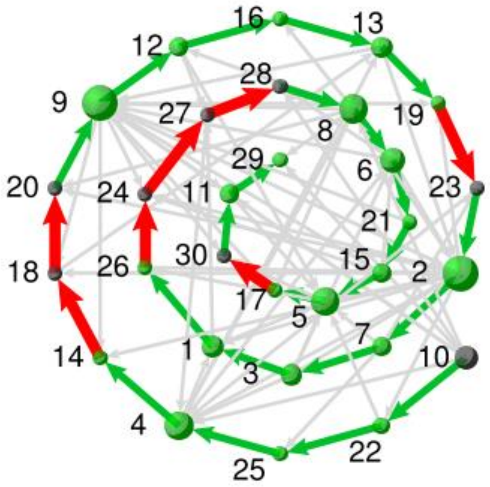
2. Purpose

3. Work

4. Conclusion



(a)



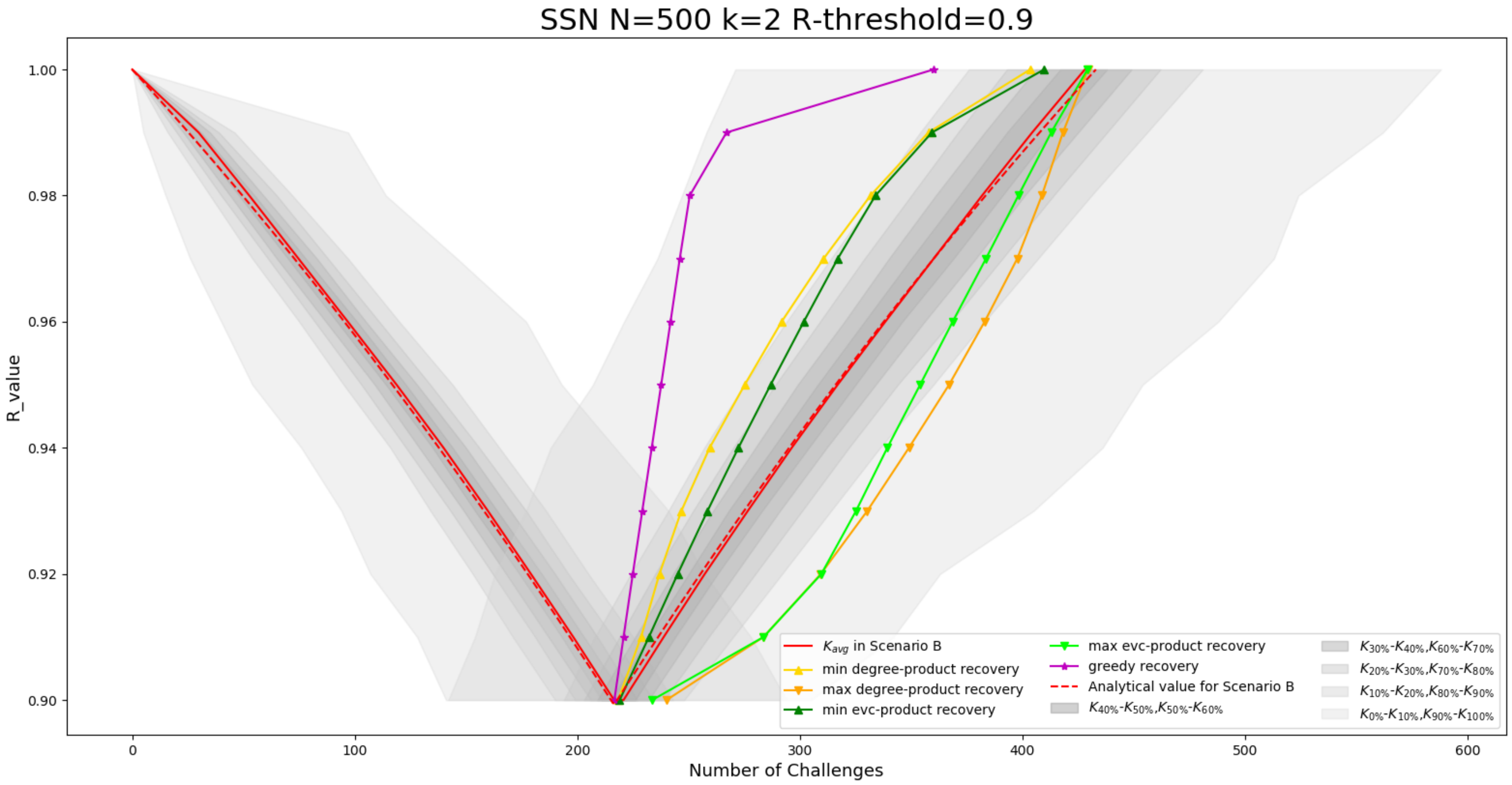
(b)

1. Finding independent matching paths
2. Ordering them
3. Linking two independent matching paths in order in each step

1. Optimizing controllability of complex networks by minimum structural perturbations (Wang,2012)

Recover strategies-scenario B

1. Introduction
2. Purpose
3. Work
4. Conclusion



TU Delft

Conclusion

Conclusion

1. Introduction

2. Purpose

3. Work

4. Conclusion

- ✓ Analytical express n_D after removing/adding links
- ✓ Analytical express n_D during attack/recovery process in two different scenarios
- ✓ Connect Strategy in Scenario A & Greedy Strategy in Scenario B

TU Delft

Thanks!

Q&A