Epidemic Spreading with Heterogeneous Infection Rates

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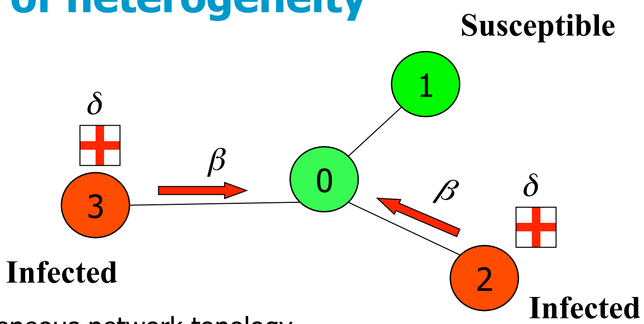
SIS Epidemic Spreading on a Network
Types of heterogeneity



- Heterogeneous network topology
- Heterogeneous infection rates:

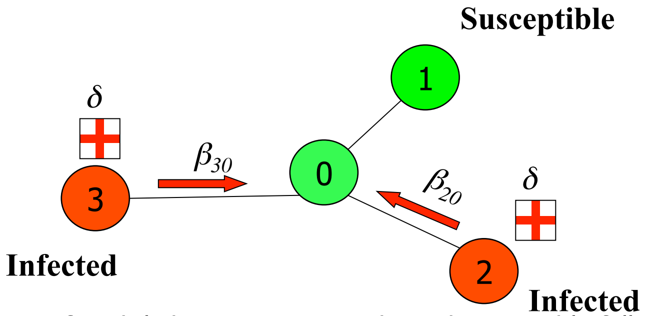
a) $\beta_{ij} = \beta_{ji}$ is an independently identically distributed random variable

b) Correlated with topology $\beta_{ij} = \beta_{ji} \sim (d_i d_j)^{\alpha}$

- Heterogeneous recovery rates allocation to reduce prevalence

B. Qu and H. Wang, SIS Epidemic Spreading with Heterogeneous Infection Rates, *IEEE Transactions on Network Science and Engineering* , 99 pp.1-1, 2017.

SIS Epidemic Spreading with i.i.d. Infection Rates



The infection rate of each link $\beta_{ij} = \beta_{ji}$ is an i.i.d. random variable following a given distribution. Three classes of distributions are considered.

Effective infection rate $\tau = E[B]/\delta = \mathbf{1}/\delta$

Objective: influence of i.i.d. infection rates (variance, higher order moments) on the average fraction of infected nodes in the metastable state.

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i.i.d. infection rates

$\tau = E[\beta_{ij}]/\delta = \mathbf{1}/\delta$

- Log-normal distribution $B \sim Log\text{-}\mathcal{N}(\beta;\mu,\sigma)$

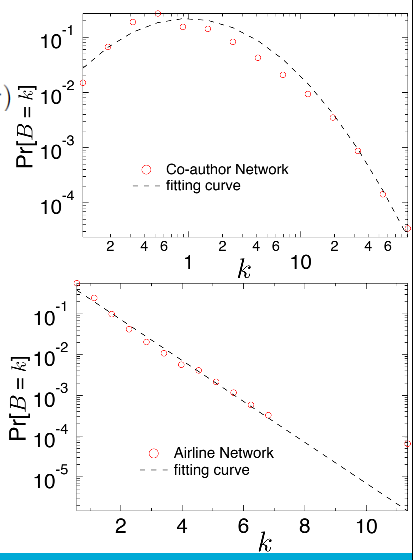
$$f_B(\beta;\mu,\sigma) = \frac{1}{\beta\sigma\sqrt{2\pi}} exp\left(-\frac{(\ln\beta-\mu)^2}{(2\sigma^2)}\right)$$

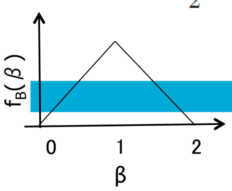
- Gamma distribution $B \sim \Gamma(\beta;k,\theta)$

$$f_B(\beta;k,\theta) = exp(-\frac{\beta}{\theta})\frac{\beta^{k-1}}{\theta^k\Gamma(k)}$$

- A symmetric distribution $B \sim SP(\beta;a,b)$

$$f_B(\beta;a,b) = \frac{b(a+1)}{2}|\beta-1|^a$$





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Simulation setup

ER and SF networks, N=10000, E[D]=4

For each of the three types of distributions,
E[*B*]=1, the variance ν is tuned over a large range by controlling the two parameters

Recovery rate δ: small and large

Continuous time simulation of the SIS process on a network to find the average fraction y_∞ of infected nodes in the meta-stable state.

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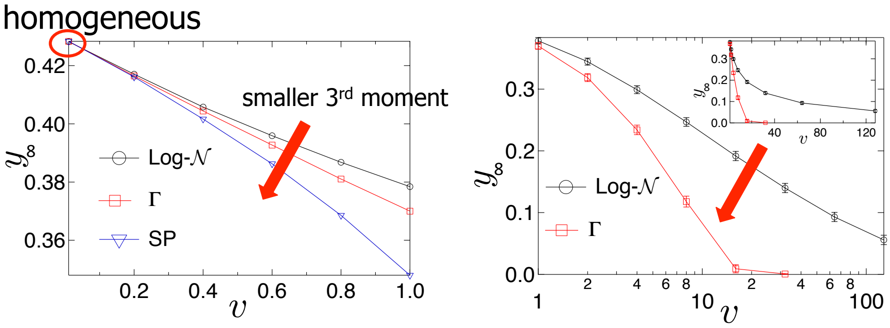
$\tau = E[B]/\delta = \mathbf{1}/2$

Small recovery rate

homogeneous



Heterogeneity i.e. variance ν ↑ prevalence y_∞ ↓

Given the variance ν, 3rd moment ↓ y_∞↓

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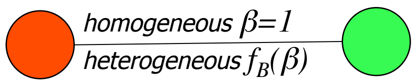
Small recovery rate

Why variance v ↑ 3rd moment ↓ lead to prevalence y_∞ ↓ ?

ρ*(T) infection probability within time T



ρ(T)

Theory: $$\rho^*(T)-\rho(T)=\sum_{n=0}^{\infty}(v_{2n}-1)\frac{T^{2n}}{(2n)!}-\sum_{n=0}^{\infty}(v_{2n+1}-1)\frac{T^{2n+1}}{(2n+1)!}>0$$

v_{2n} is the 2n-th moment of B and variance $v=v_2-1$

Local: epidemic spreads faster on average along a link in homogeneous case

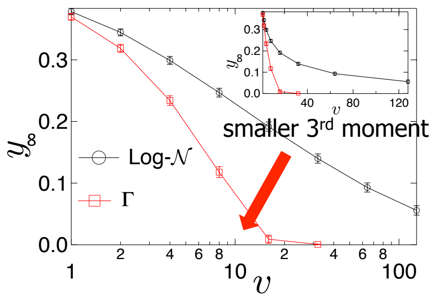
Global?

B. Qu, C. Li, P. Van Mieghem and H. Wang, Ranking of Nodal Infection Probability in Susceptible-Infected-Susceptible Epidemic, *Scientific Reports* **7**, 9233, 2017.

Small recovery rate

3rd moment ↓ lead to prevalence y_∞ ↓ ?

E[B]=1, v=16



Percentiles	$Log-\mathcal{N}$	Γ
1^{st}	0.00483	9.44×10^{-32}
2.5^{th}	0.00895	2.20×10^{-25}
5^{th}	0.0152	1.44×10^{-20}
10^{th}	0.0280	9.44×10^{-16}
25^{th}	0.0779	2.20×10^{-9}
50^{th}	0.243	1.44×10^{-4}

Network filtering

A large variance is obtained by having extremely small (large) values in gamma (Log-normal) distribution.

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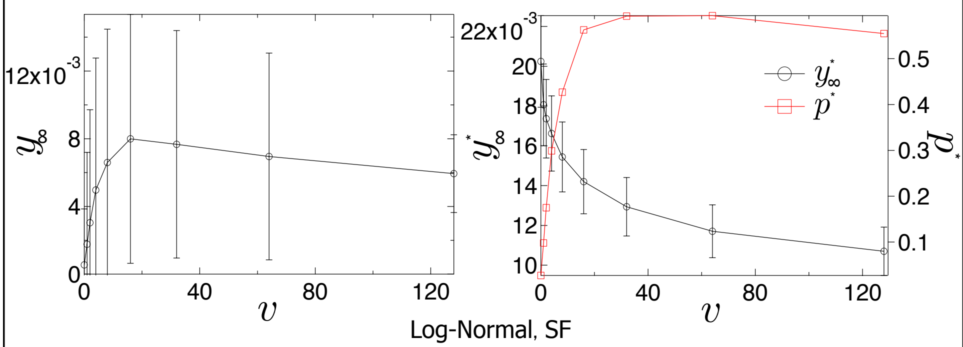
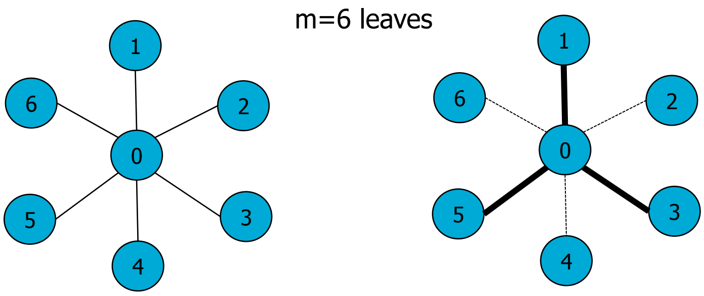
Large recovery rate

$\tau = E[B]/\delta = \mathbf{1}/20$



p*, percentage of non-zero infection realizations, may increase with v.
Links with large infection rates form a connected subgraph that allows an epidemic to spread out.

Large recovery rates- in a star



$$\delta = \sqrt{m} + \epsilon$$

$\beta = 1$

$\tau_c = \frac{1}{\sqrt{m}}$

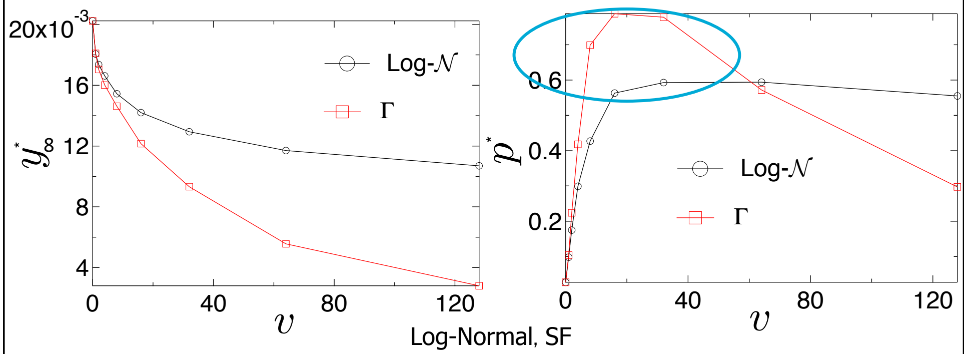
$\tau = \frac{\beta}{\delta} = \frac{1}{\sqrt{m}+\varepsilon} < \tau_c$

m/2 links with $\beta = 0$, m/2 links with $\beta = 2$

$\tau_c^* = \frac{1}{\sqrt{m/2}}$

$subgraph: \tau^* = \frac{2}{\sqrt{m}+\varepsilon} > \tau_c^*$

Large recovery rates

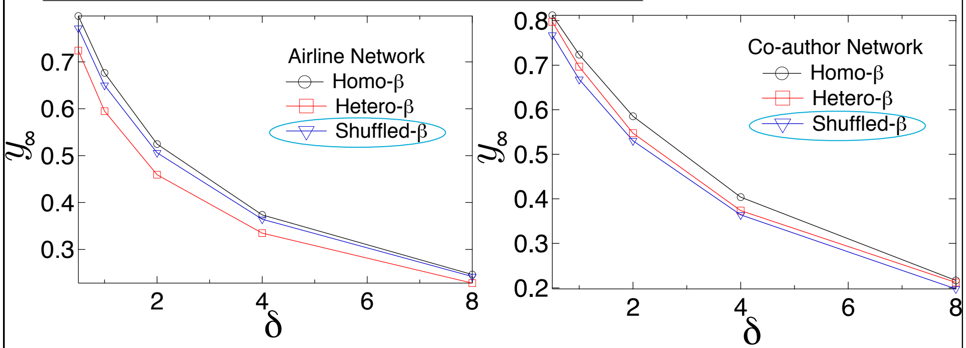


Links with large infection rates form a connected subgraph that allows an epidemic to spread out.
Gamma (log-normal) distribution leads to many large (a few extremely large) infection rates.

Real-world networks and infection rates

Name	Nodes	Links	Variance	Range
Airline	3071	15358	0.5560	[0.2383, 11.0626]
Co-author	39577	175692	3.0566	[0.0678, 90.4625]

Scale-free



Small recovery rate: variance ν ↑ prevalence y_∞ ↓

Real-world networks and infection rates

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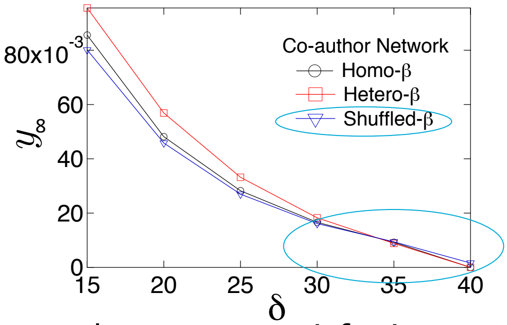

Large recovery rate: heterogeneous infection rate may contribute to the survival of an epidemic

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Conclusion and discussion

We illustrate with simulation, theoretical analysis and physical interpretations effects of heterogeneous infection rates on epidemic spreading around and above epidemic threshold.

What if the infection rate of a link is correlated with e.g. the degrees of its two end nodes?

How accurate is the mean-field approximation in these two cases?

What if the infection process along each line is non-Markovian?

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Thank You
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