Indirect influence in social networks as an induced percolation phenomenon

Xiangrong Wang
Southern University of Science and Technology
Peng Cheng National Laboratory, Shenzhen, China

Joint with
Jiarong Xie, Ling Feng, Wenyuan Liu, Jin-Hua Zhao, Yamir Moreno and Yanqing Hu

SUSTech
Southern University of Science and Technology

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- Methods
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- Take-away message

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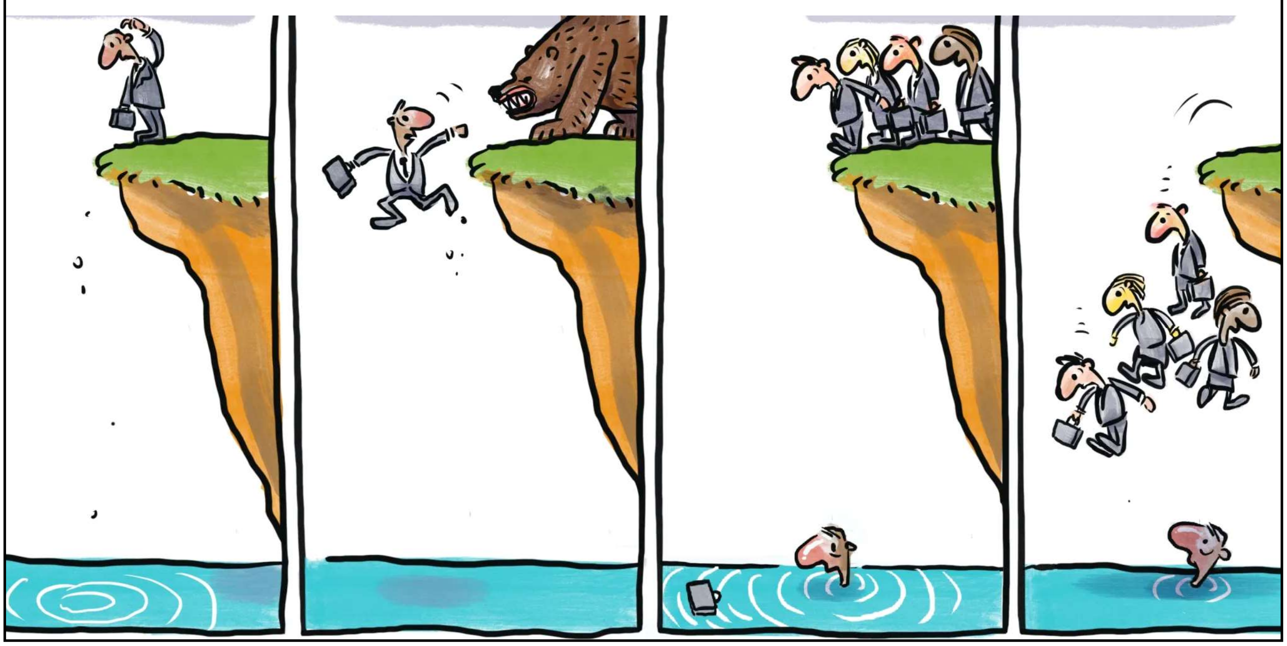
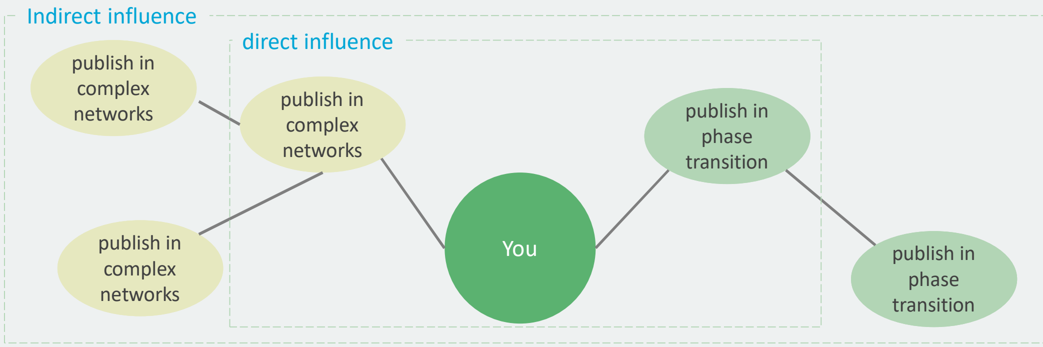
Motivation- behaviors can be contagious in social networks



Motivation- behaviors can be contagious in social networks

Publishing behavior in collaboration networks



complex networks OR phase transition?

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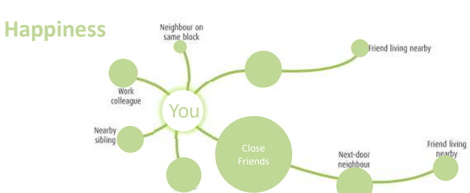
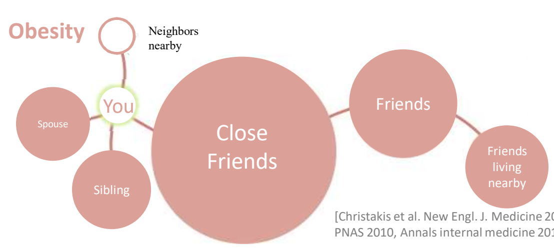
Motivation- Indirect influence in behavior contagion

- Indirect influence in behavior contagion from social experiments
  - Obesity experiment lasting 32 years
    - person-to-person spread of obesity
    - extended to three degrees of separation
  - Repeatedly found in behaviors of happiness, smoking, drug, alcohol, loneliness, among others.





[Christakis et al. New Engl. J. Medicine 2007, PNAS 2010, Annals internal medicine 2010]

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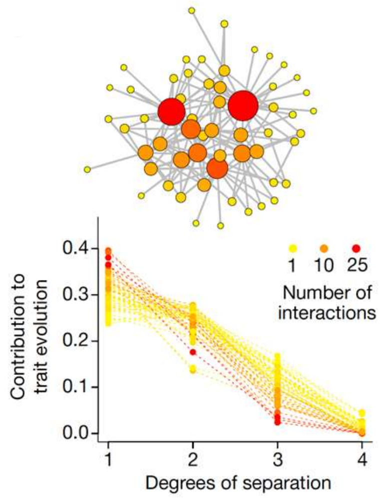
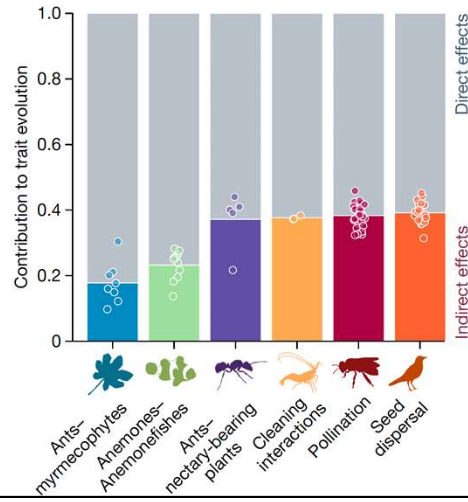
Motivation- Indirect influence in behavior contagion

- Indirect influence observed in ecological trait evolution.





[Guimaraes et al. Nature, 2017]

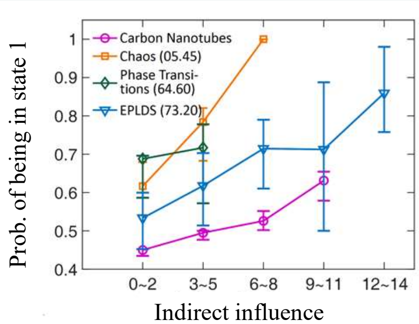
Motivation- Indirect influence in behavior contagion

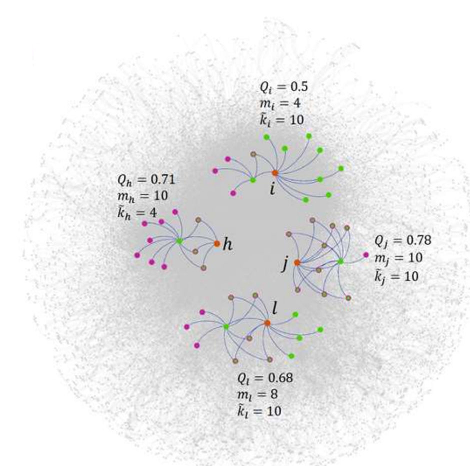
- Indirect influence observed in scientific collaboration

Scientists behaviors:

State 1 - publish in Com. Net.

State 0 - publish in Pha. Trans.





Research question

What are the potential underpinning mechanisms for indirect influence in behavior contagion,

so that we can propose models to mimic the contagion process?

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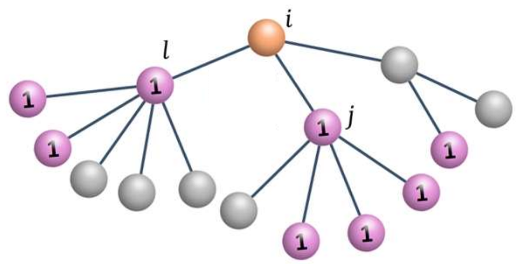
Indirect influence as an induced percolation

- **Model of Induced percolation**:



Induced index m_i :

$$m_i = \begin{cases} \text{Max indirect (second) neighbors in state 1} \\ \text{of those} \\ \text{Direct neighbors in state 1} \end{cases}$$

$m_i = 3$

$k_i = 2, d_i = 6$

Direct influence as comparison

k-core index k_i: directed neighbors in state 1

degree index d_i: second neighbors in state 1

Indirect influence as an induced percolation

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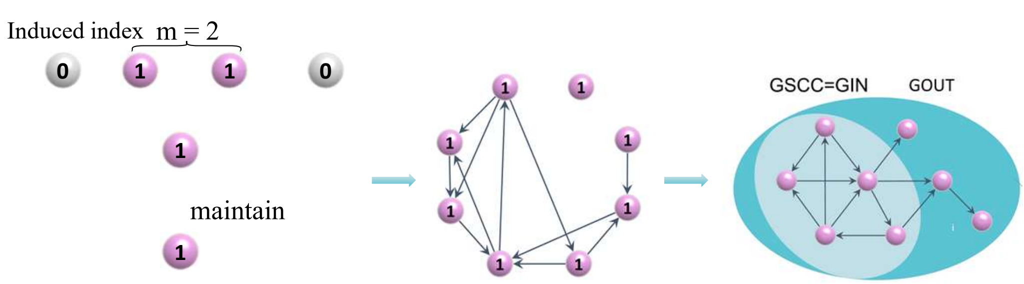
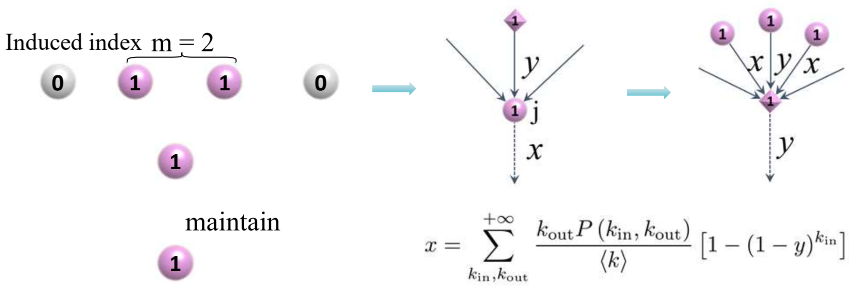


Order parameter GOUT: corresponds to the largest spreading coverage.

Indirect influence as an induced percolation

- **Model of Induced percolation**:



x: Prob. of starting node in state 1
y: Prob. of an end node in state 1
P_∞: Order parameter GOUT

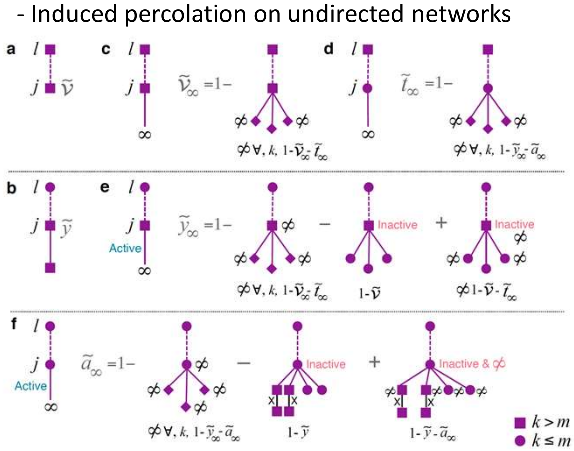
$$x=\sum_{k_{\rm in},k_{\rm out}}^{+\infty}\frac{k_{\rm out}P\left(k_{\rm in},k_{\rm out}\right)}{\langle k\rangle}\left[1-(1-y)^{k_{\rm in}}\right]$$

$$y=\sum_{k_{\rm in},k_{\rm out}}^{+\infty}\frac{k_{\rm out}P(k_{\rm in},k_{\rm out})}{\langle k\rangle}\sum_{s=m}^{k_{\rm in}}\binom{k_{\rm in}}{s}x^s(1-x)^{k_{\rm in}-s}\left[1-\left(1-\frac{y}{x}\right)^s\right]$$

$$P_\infty=\sum_{k_{\rm in},k_{\rm out}}^{+\infty}P\left(k_{\rm in},k_{\rm out}\right)\left[1-(1-y)^{k_{\rm in}}\right]$$

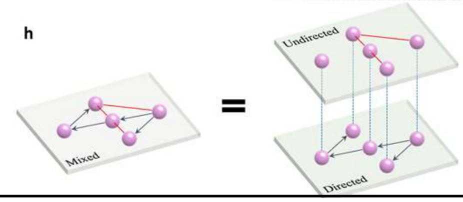
Indirect influence as an induced percolation

- **Model of Induced percolation**:
  - Induced percolation on undirected networks



- Induced percolation on mixed networks

Conditional probability	Link type	Conditional event	Occurring event		Relation
y	Directed (j,l)	None	$j \to l$		
a			j is active but $j \nrightarrow l$		
x			j is active		$x = y + a$
$\tilde{v}$	Undirected $\{j,l\}$	$l \to j$	$j \to l$		
$\tilde{y}$		$l \nrightarrow j$	$j \to l$		
$\tilde{a}$		$l \nrightarrow j$	j is active but $j \nrightarrow l$		
$\tilde{x}$		$l \nrightarrow j$	j is active		$\tilde{x} = \tilde{y} + \tilde{a}$
y_∞	Directed (j,l)	None	(i) $j \to l$	and (ii) l connects to GOUT via j	
a_∞			(i) $j \nrightarrow l$		
x_∞			(i) j is active		$x_\infty = y_\infty + a_\infty$
$\tilde{t}_\infty$	Undirected $\{j,l\}$	$l \to j$	(i) $j \nrightarrow l$		
$\tilde{v}_\infty$		$l \to j$	(i) $j \to l$		
$\tilde{y}_\infty$		$l \nrightarrow j$	(i) $j \to l$		
$\tilde{a}_\infty$		$l \nrightarrow j$	(i) $j \nrightarrow l$		
$\tilde{x}_\infty$		$l \nrightarrow j$	(i) j is active		$\tilde{x}_\infty = \tilde{y}_\infty + \tilde{a}_\infty$



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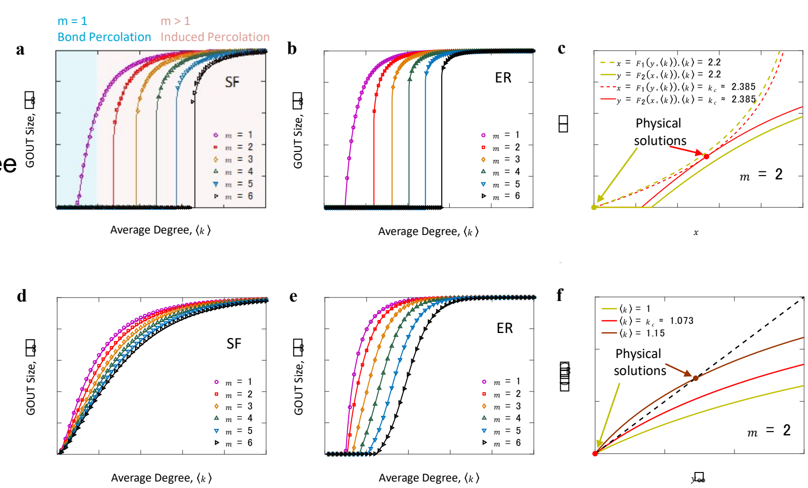
Rich critical behaviors induced by indirect influence

- Order parameter GOUT on directed nets
  - $\langle k \rangle$: average degree
  - m: induced index

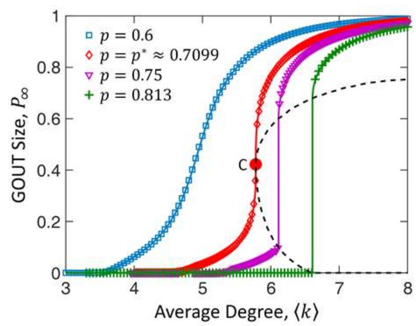


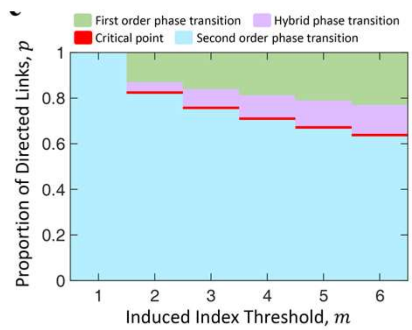
- GOUT on undirected nets:

J Xie, X Wang, L Feng, JH Zhao, Y Moreno, Y Hu, Induced Percolation on Networked Systems , PNAS, 119(9), 2022.

Rich critical behaviors induced by indirect influence

- GOUT on mixed nets:
  - $\langle k \rangle$: average degree
  - m: induced index
  - p: proportion of directed links





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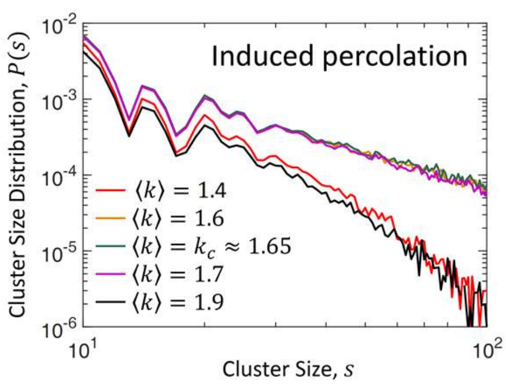
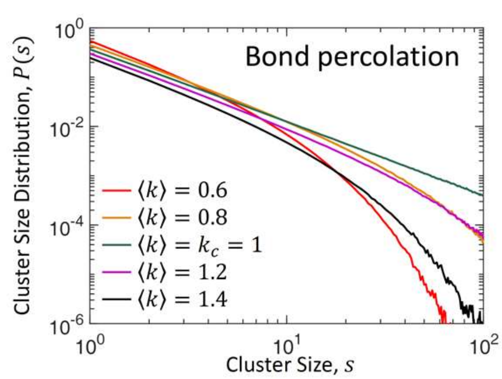
Rich critical behaviors induced by indirect influence

- Size distribution $P(s)$ of small clusters at the critical point of induced percolation ($m = 4$) on undirected networks.





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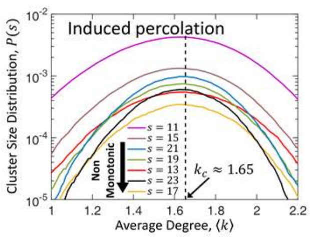
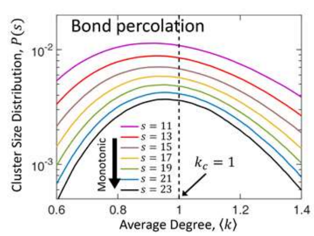
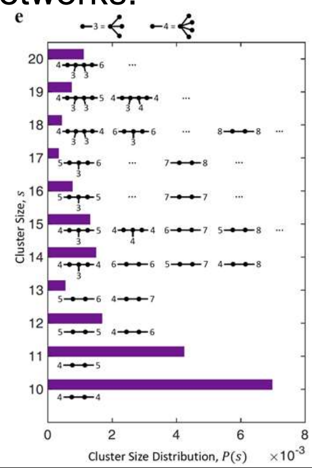
Rich critical behaviors induced by indirect influence

- Size distribution $P(s)$ of small clusters at the critical point of induced percolation ($m = 4$) on undirected networks.







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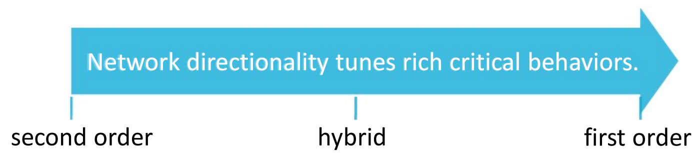
Take-away message

- Indirect influence in social networks as an induced percolation phenomenon
- Induced percolation leads to rich critical behaviors depending on a single network parameter.



Feel free to contact me at:

E: xiangrongwang88@gmail.com or W: https://xiangrongwang.github.io/