Delft-Girona

October 12th, 2022
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Relevant metrics to explain network robustness

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Introduction

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Objectives

- Classify networks depending on which type of attack is most vulnerable.
- Explore the metrics that best explain their classification.
- Design an algorithm to improve the robustness of networks.

Definition

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Robustness is defined as the characteristic of a network that reflects its ability to continue to function properly despite failures or attacks.

State of the art

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2013

Trajanovski and others proposed an original way of calculating the robustness with the weighted sum of several metrics:

$$R = \sum_{n}^{k=1} s_k t_k$$

2014

Manzano and others proposed the idea of using a dimensionality reduction algorithm, the PCA, to find the weights of the metrics.

2018

Marzo and others proposed a list of ten recommended metrics to compute the robustness of a network:

- Average Nodal Degree
- Efficiency
- Largest Eigenvalue
- Largest Connected Component
- Average Two Terminal Reliability
- Algebraic Connectivity
- Natural Connectivity
- Edge Betweenness Centrality
- Closeness Centrality
- Eigenvector Centrality

Computation

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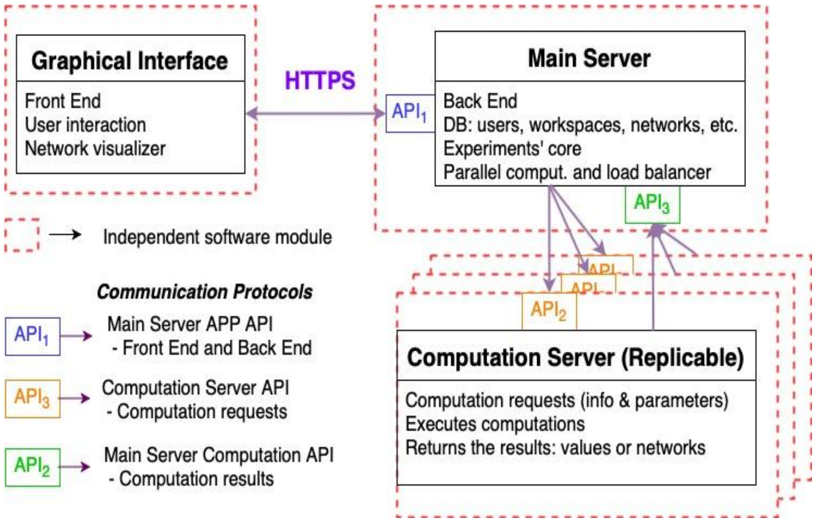
1. The original network is attacked and elements are removed sequentially up to a value P. This is repeated M times which generates M x P networks.
2. For each network generated in the previous point, the value of the ten recommended metrics is calculated by generating a matrix of (MxP) x10.
3. PCA is applied to the matrix of metrics and the first principal component is chosen.
4. The weights of the first principal component are normalized so that the robustness of the original network is 1.
5. All the rows of the matrix of metrics are multiplied by the normalized weights obtained in the previous point. This is how the robustness of each $network_{mp}$ is found.
6. The average robustness of the network is calculated from the vector of robustness in point 5.

NRS and Topology Zoo

The Network Research Simulator (NRS) is a development tool of the BCDS group that allows the visualization and the computation of the robustness of networks.



The Topology Zoo is a public repository of telecommunication networks. We picked 205 of the 261 files that meet the following conditions:

- More than 15 nodes.
- The networks are initially connected.

Attacks

We remove nodes with the next four metrics:

Nodal Degree

Prioritize the nodes with the higher nodal degree.

Betweenness Centrality

Prioritize the nodes with the higher number of the shortest paths that pass by them.

Closeness Centrality

Prioritize the nodes that are closer to the other nodes considering the shortest paths.

Eigenvector Centrality

Prioritize the nodes by importance, meaning, those nodes that are connected to others that have a high score of the metric.

Attacks

The four strategies we use to remove the nodes are:

Nodal Degree

Prioritize the nodes with the higher nodal degree.

Betweenness Centrality

Prioritize the nodes with the higher number of the shortest paths that pass by them.

SPOILER: It's the worst

Closeness Centrality

Prioritize the nodes that are closer to the other nodes considering the shortest paths.

Eigenvector Centrality

Prioritize the nodes by importance, meaning, those nodes that are connected to others that have a high score of the metric.

Previous study

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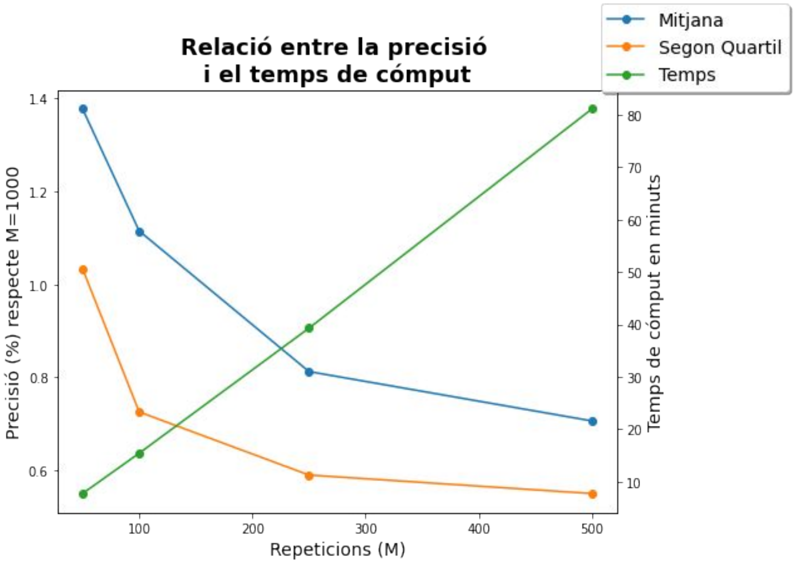
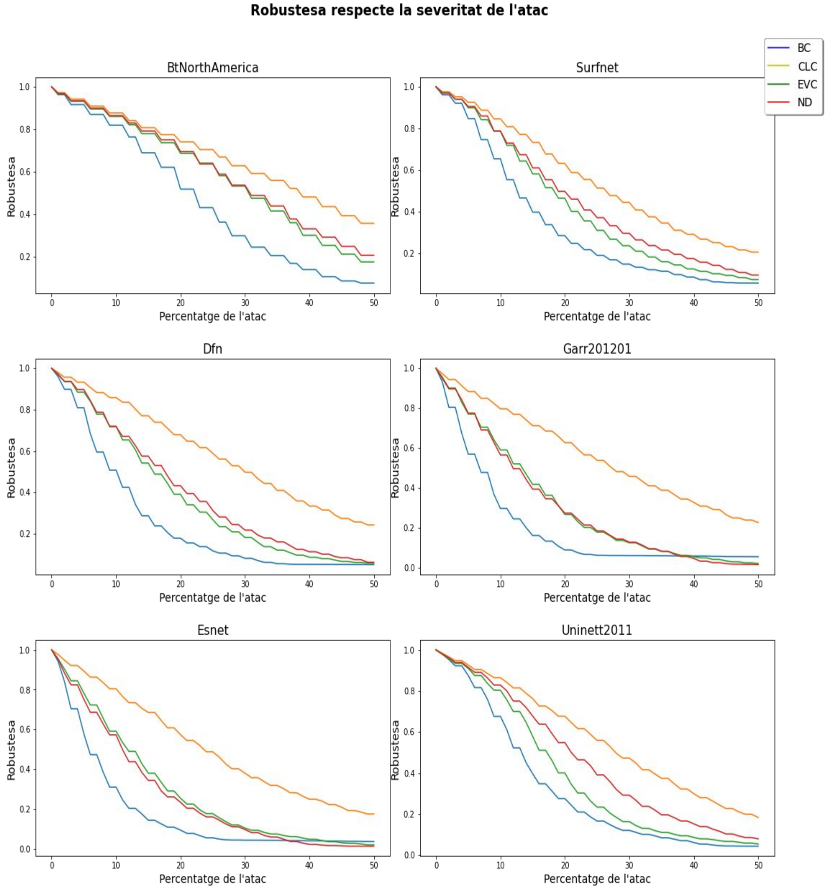
The first step was to find the most suitable values of the M (number of attacks) and the P (depth of the attack) for the generation of the dataset.





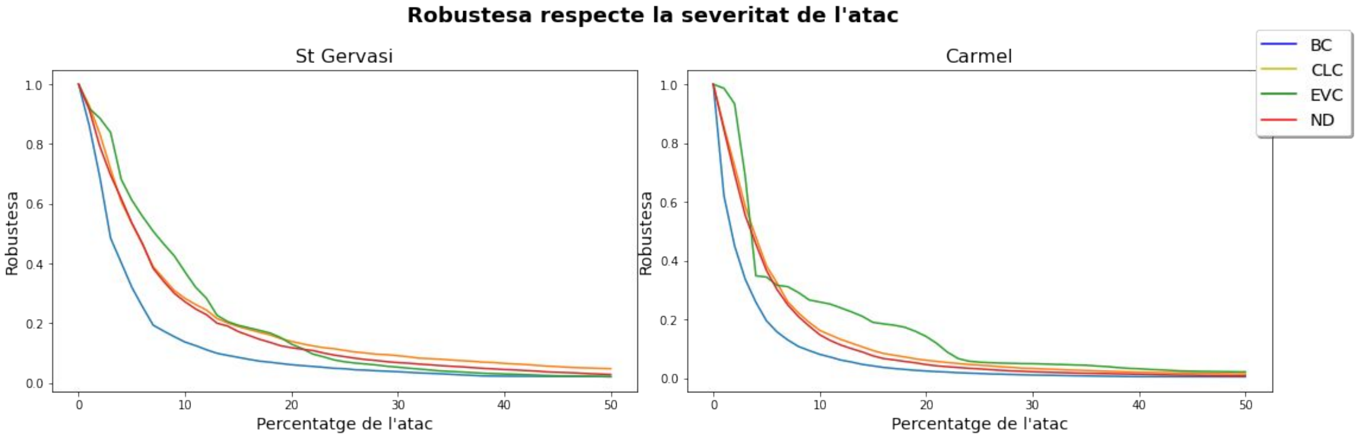
Initial exploration

Only three networks from Topology Zoo have worse robustness to a different attack than Betweenness Centrality, but the difference is negligible.

So, what now?

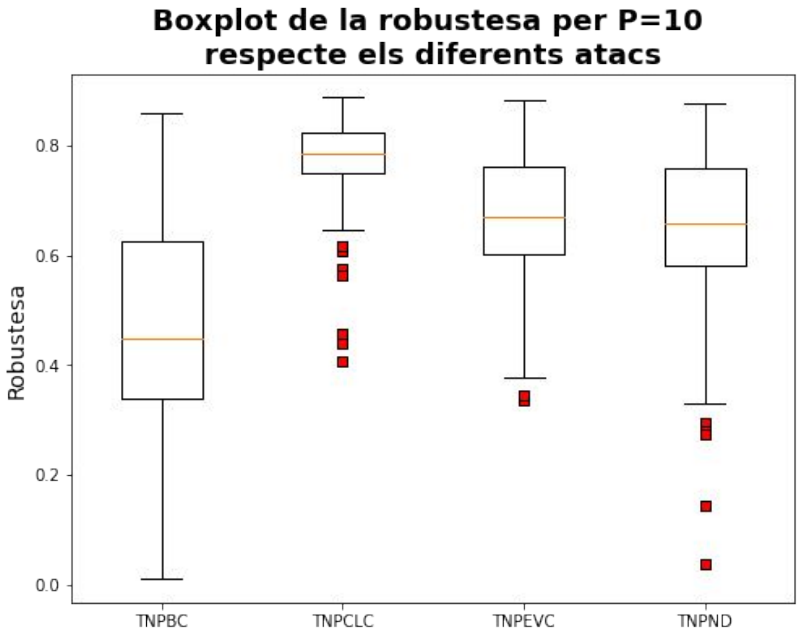
- Do we add other topologies?
- Do we change the approach?

Network	Robustness for BC	Difference
Belnet2009	0.802240	0.000724
Belnet2010	0.598901	0.003045
Nextgen	0.829055	0.007802



Network classification

Finally, we decided to classify the network depending on which quartile the robustness for P=10 is because each class will have the same amount of samples, and the range is close to all possible values.



Some interesting metrics

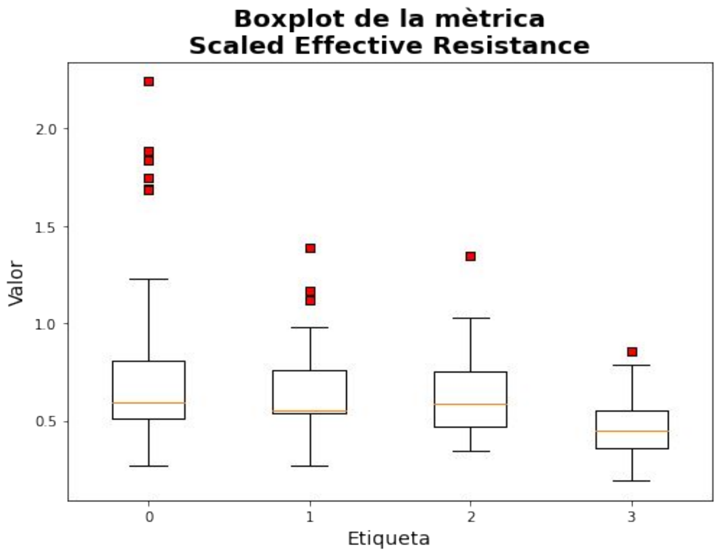
- **Heterogeneity**: It is the standard deviation of the nodal degree divided by the average nodal degree.
- **Effective Resistance**: This considers the graph an electrical circuit, where an edge is a resistor of 1Ω. ER is the sum of the effective resistances over all pairs of vertices.
- **Clustering Coefficient:** CC measures the probability that the adjacent nodes of a node are connected (among them). CC captures the presence of triangles and compares it to the number of connected triplets.
- **Node Betweenness Centrality:** This measures the fraction of shortest paths that pass through a given node, averaged over all pairs of nodes in the network.
- **Closeness Centrality:** This measures the degree to which a node is close to other nodes on average considering the shortest paths (per node).

Effective Resistance

The smaller the value of the **Effective Resistance**, the better because it means that the minimum paths are shorter, and there are more of them in parallel.

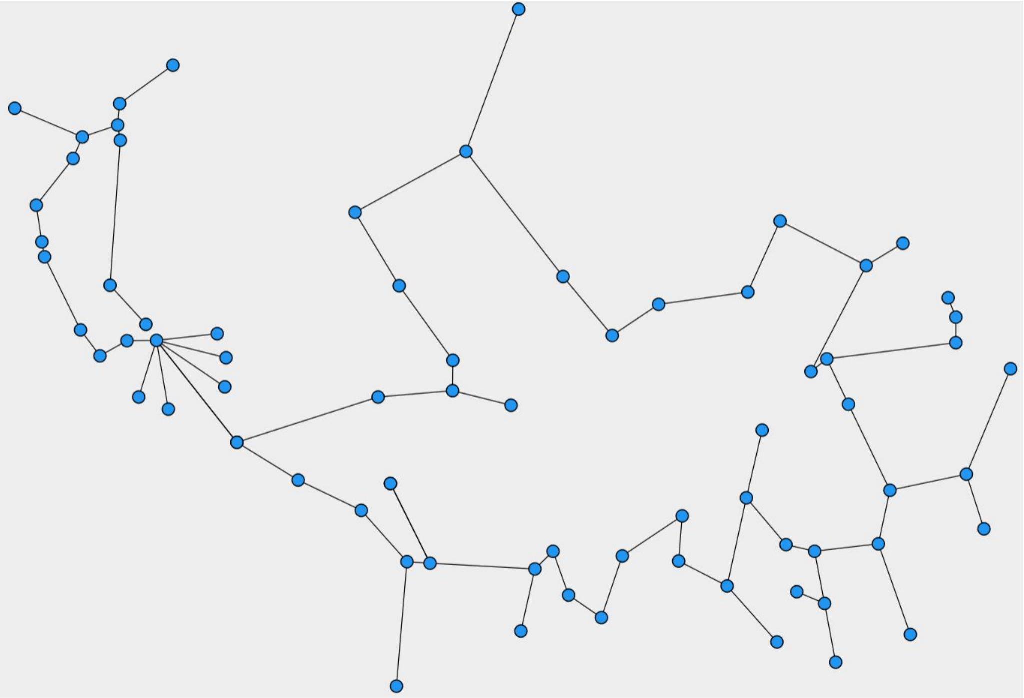
Problem: it is a metric that scales enormously with the number of nodes. First, we have to find a way to normalize between different networks.

Solution: ER is the sum of the effective resistances over **all pairs of vertices.**



Network	NN	AND	SEFF
Syringa	74	2	2.24
VtlWavenet2011	92	2.08	1.883

Effective Resistance



Syringa

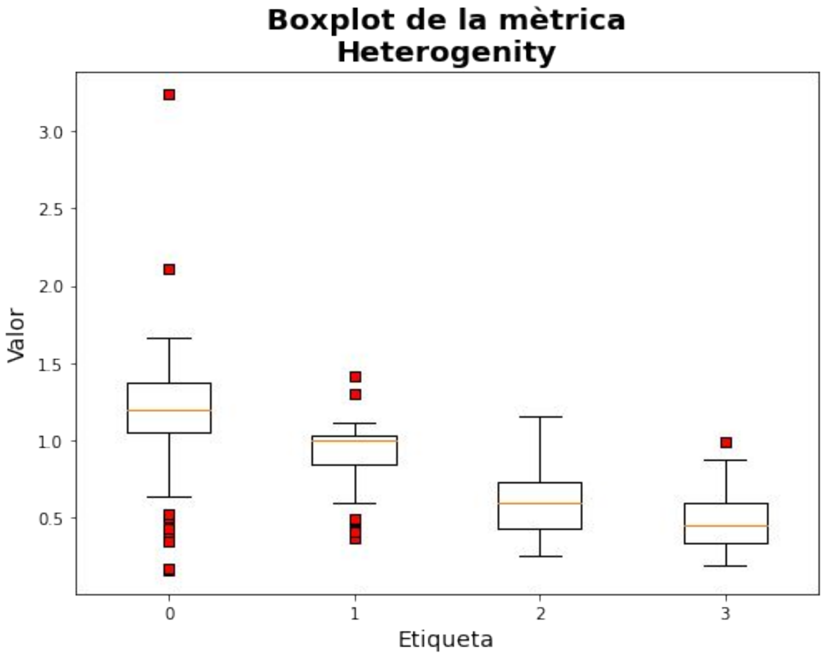
VtlWavenet2011

Heterogeneity

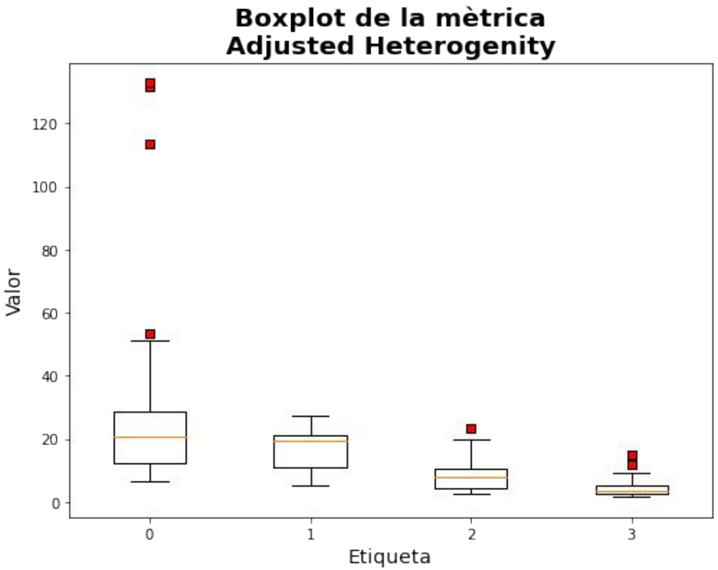
Remainder: the **heterogeneity** is the standard deviation of the nodal degree divided by the average nodal degree.

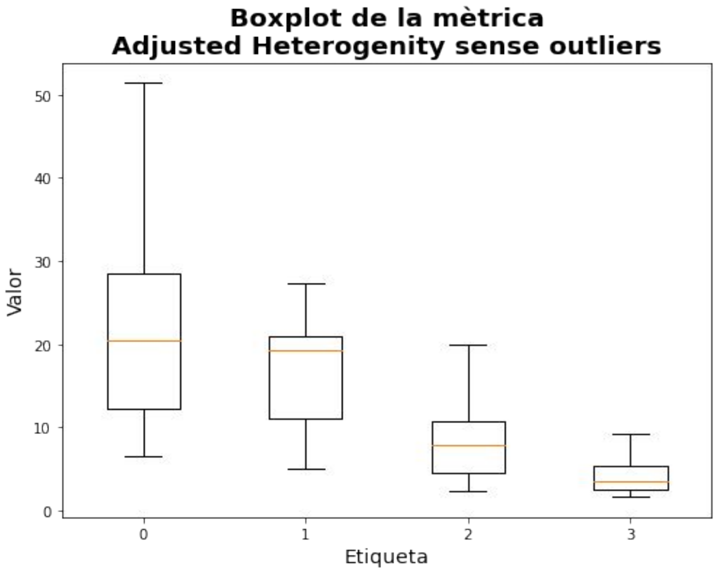
It indicates how regular the graph is.

Network	NN	AND	HET
VtlWavenet2008	88	2.09	0.156
VtlWavenet2011	92	2.08	0.168
UsCarrier	158	2.39	0.342
Kdl	754	2.38	0.358
Cogentco	197	2.48	0.427



Adjusted Heterogeneity



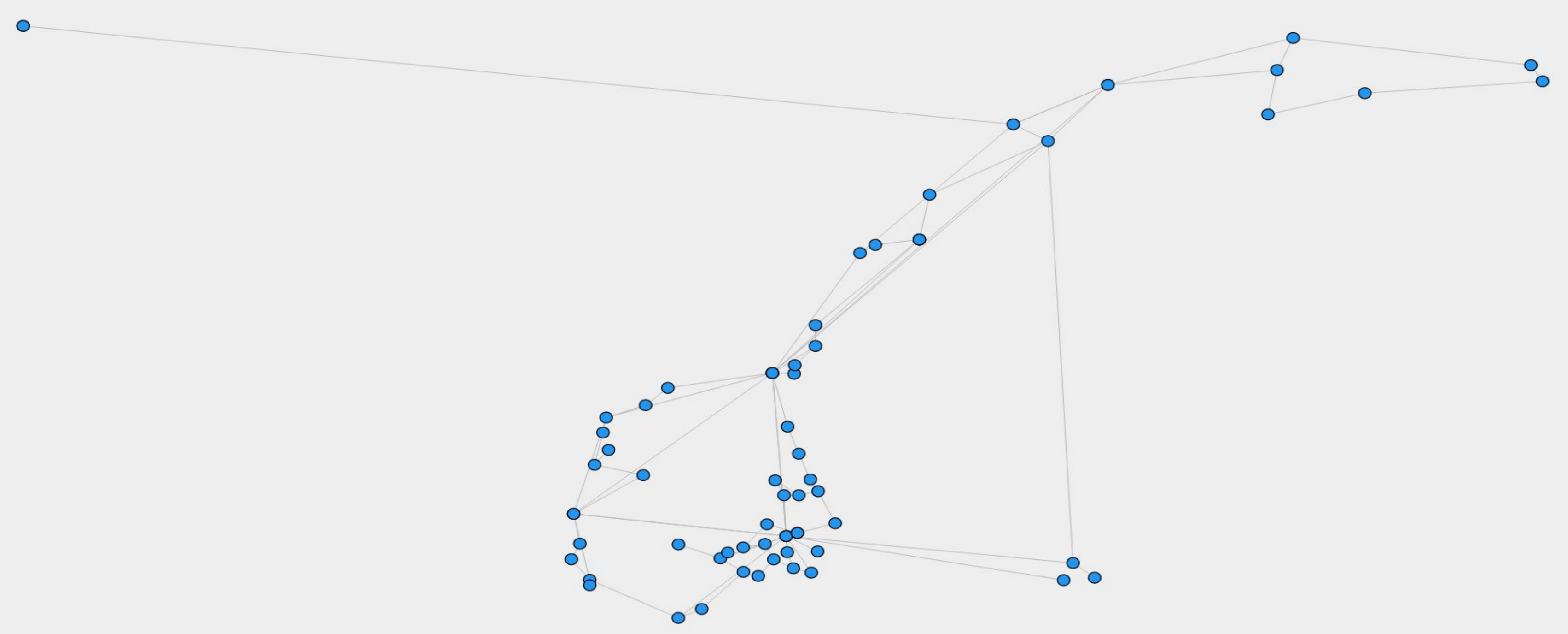


Network	NN	AND	HET	AHET
Ulaknet	82	2	3.23	132.73
Pern	127	2.03	2.10	131.64
Kdl	754	2.38	0.36	113.37

Network	NN	AND	HET	AHET
Uninett2011	69	2.84	0.61	14.85
Uunet	49	3.42	0.83	11.94
PionierL3	38	2.73	0.66	9.18

Adjusted Heterogeneity

Uninnett2011 is a Norwegian research network, and it's harder to explain why it has great robustness.



Algorithm

This algorithm is inspired by the Uninett2011 network that we just saw at the NRS. Unlike similar algorithms it doesn't work by adding new edges but, instead, by adding new nodes at the weakest point of the network.

1. Identify the nodes that, when removed, split the network into several components (critical nodes).
2. Duplicate or triplicate each critical node and add a link to connect them.
3. Identify the different neighbours of the original node belonging to each component and split the links between the new nodes.
4. Add if necessary new edges to the new nodes.

Conclusions and Future work

- It's not trivial and time-consuming to compute the robustness of a network, but it's also complicated to explain the robustness with a single metric.
- Most metrics are impossible to compare between completely unrelated networks, such as Largest Eigenvalue.
- The algorithm to improve the robustness of networks is still in the early stage of development. We just have a rough idea, but we believe it has good potential.

- Do a small experiment in which we improve the robustness of a small network manually to verify if it's a valid approach.
- Implement new metrics like Kemeny constant.

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Thank you for your attention.

Any questions?

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