Topological-Temporal Properties of Evolving Networks

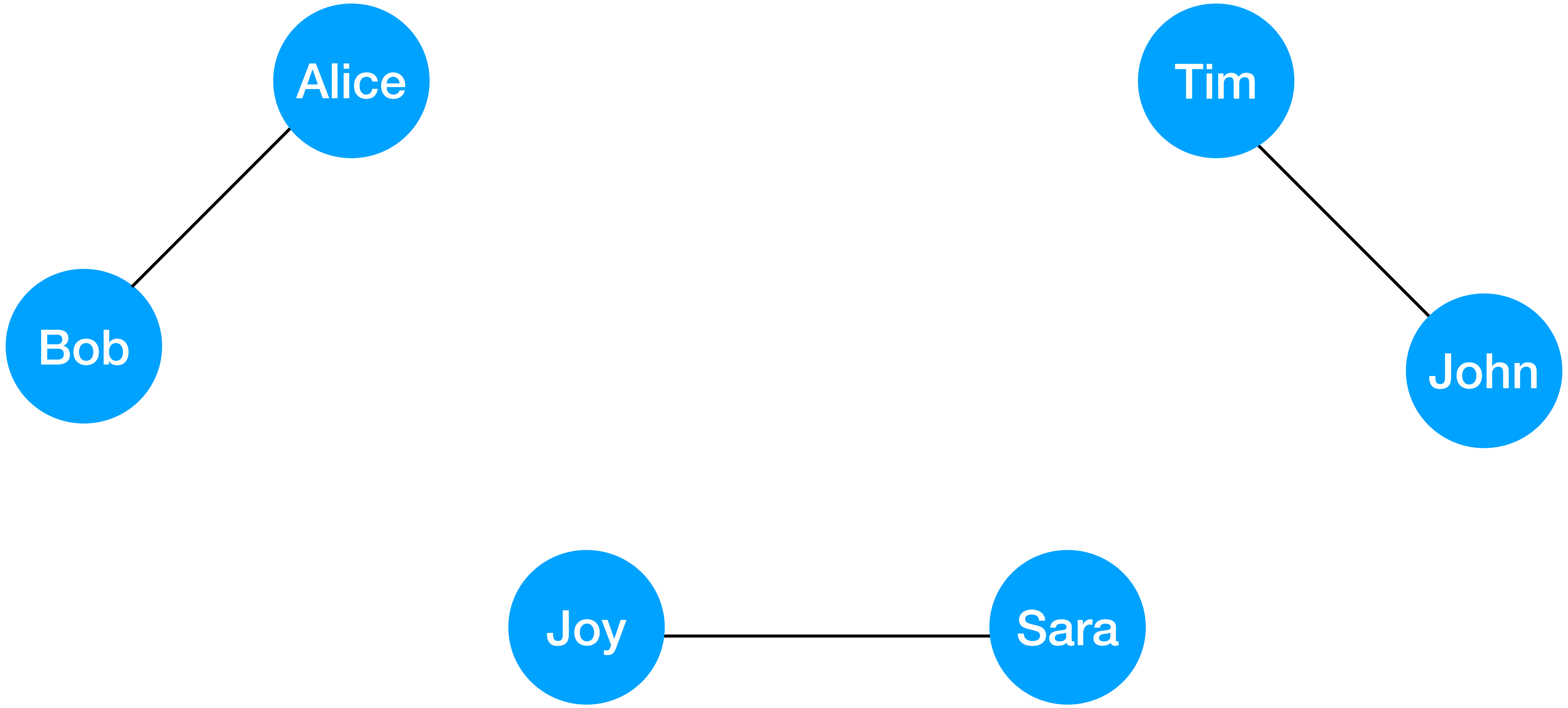
Alberto Ceria, Shlomo Havlin, Alan Hanjalic, and Huijuan Wang

Overview

- Introduction & Motivation
- Definitions & Results
- Conclusions

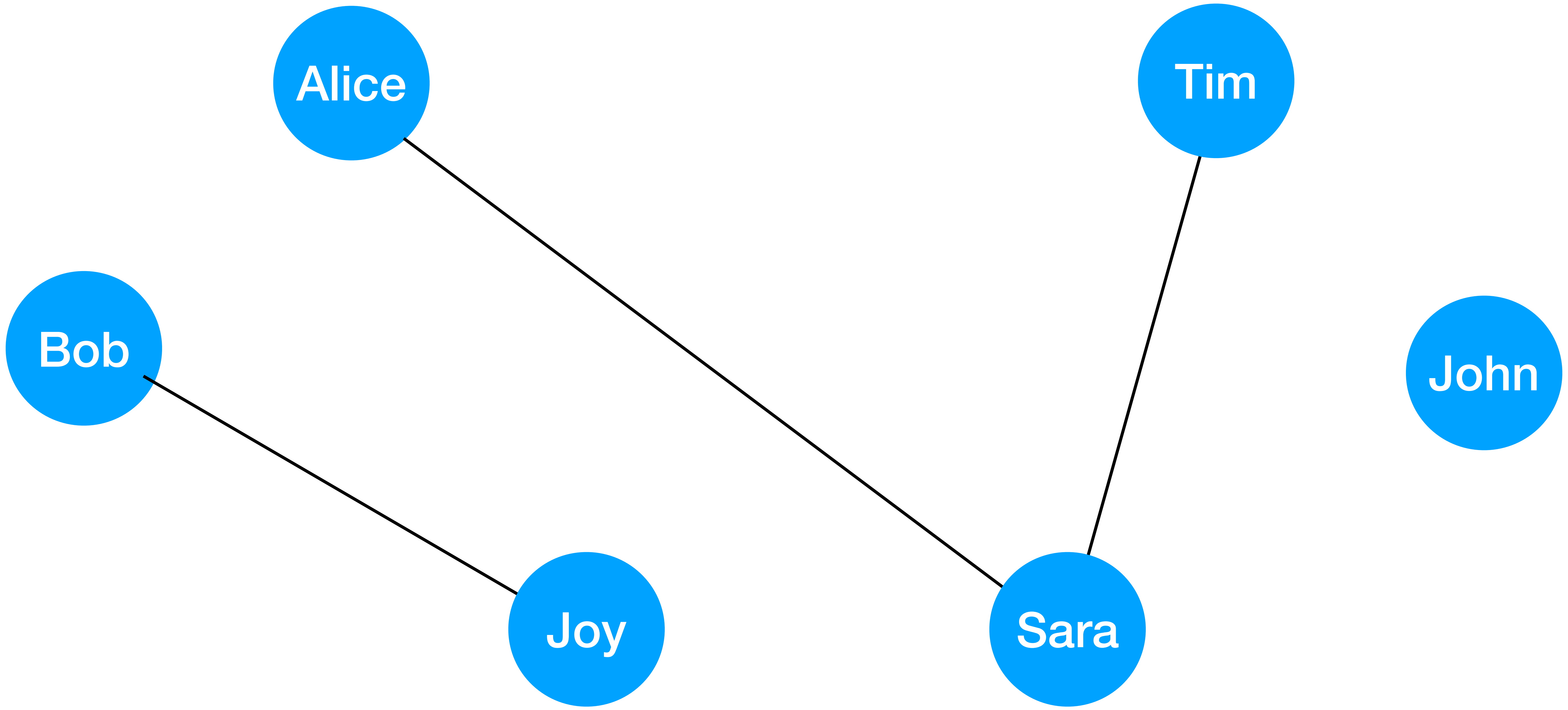
Evolving networks?

h 18:00



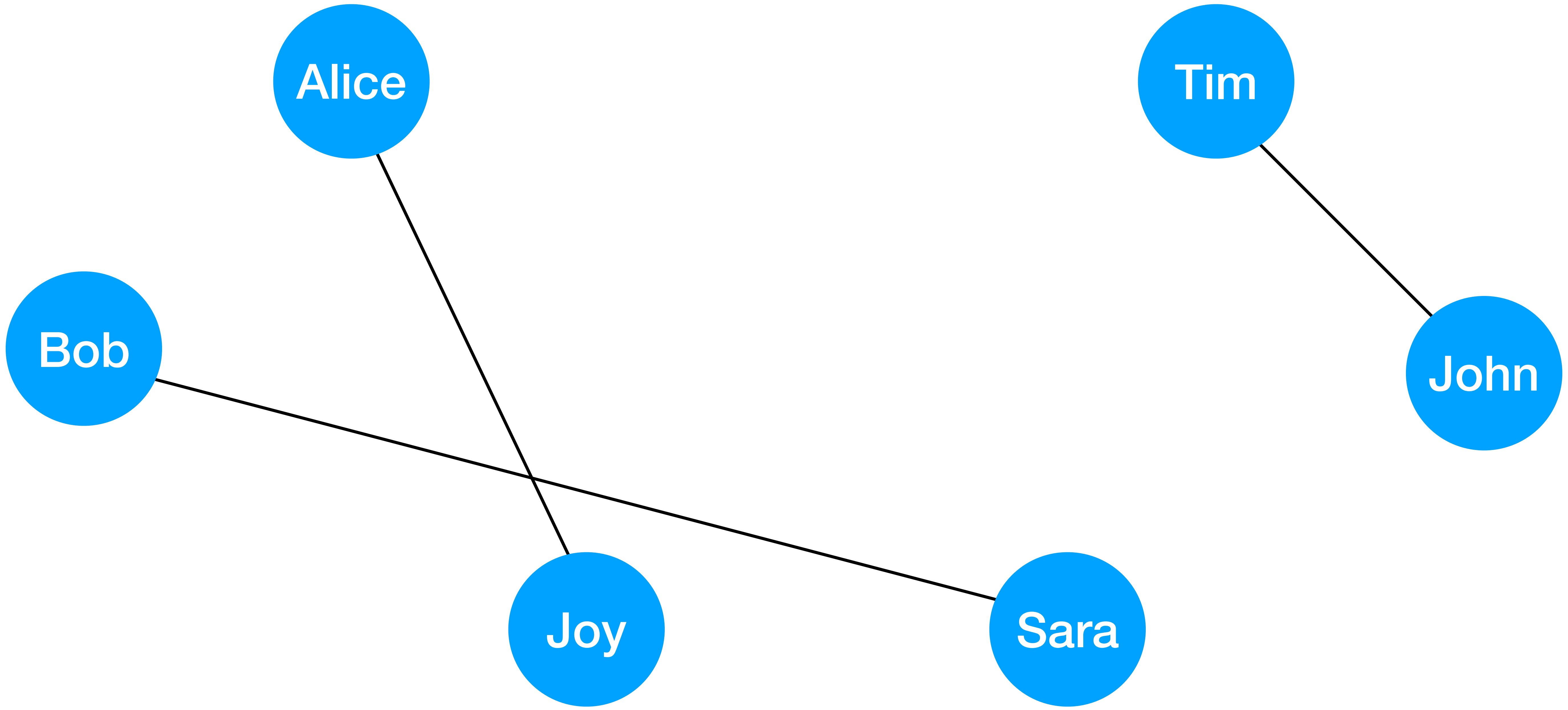
Evolving networks?

h 18:30



Evolving networks?

h 19:00



Why do we characterise evolving networks?

- **Comparing** different networks
- Detecting properties that can influence **dynamic processes** unfolding on evolving networks
- **Guiding the development** of improved **evolving network models**

How do we characterise evolving networks?

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How do we characterise evolving networks?

- Characterization of properties of (weighted) time-aggregated topology

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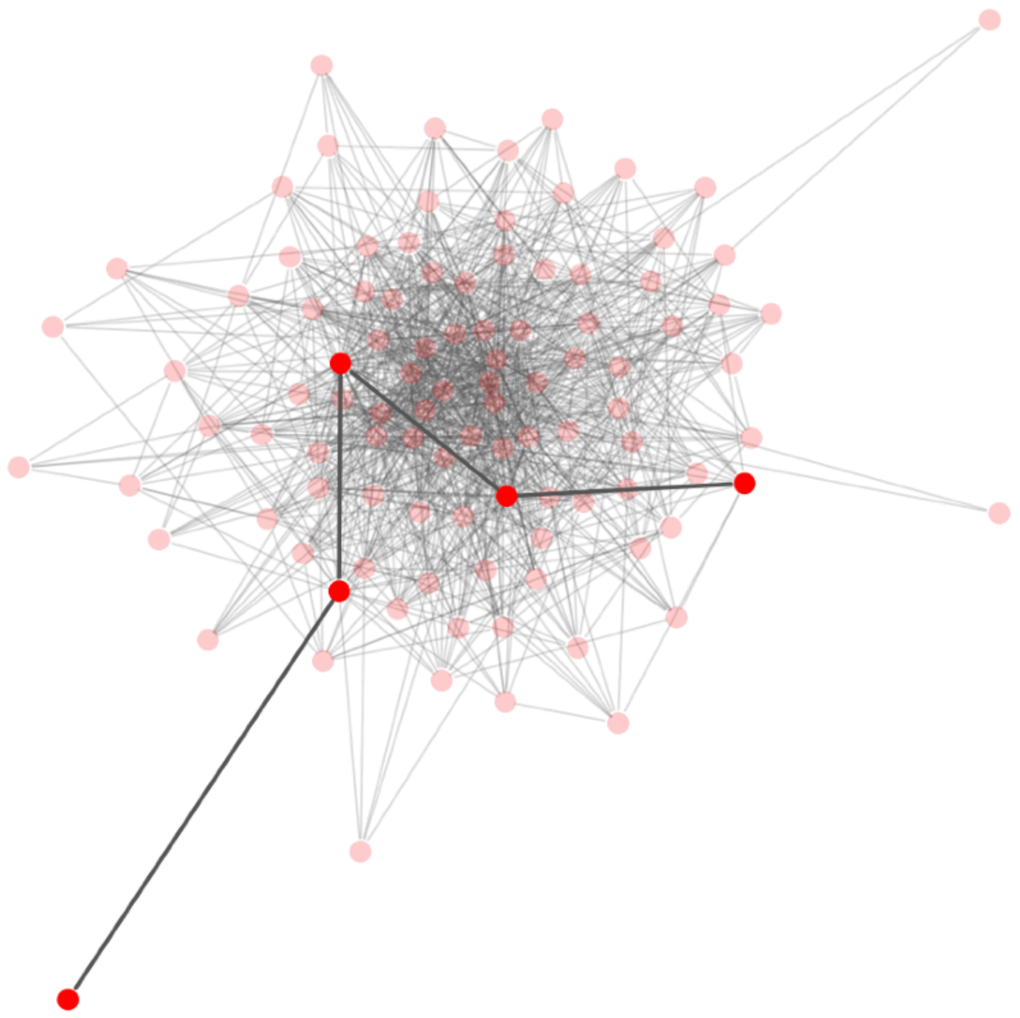

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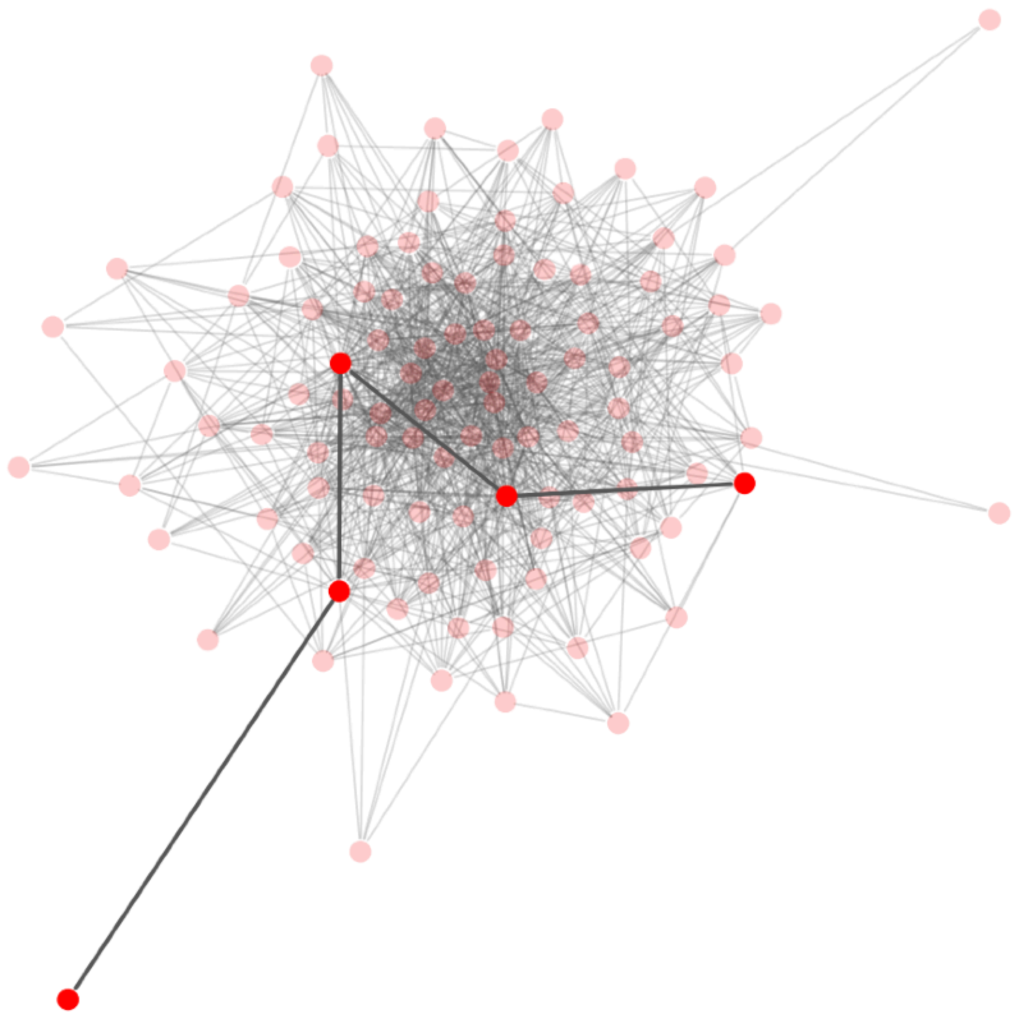

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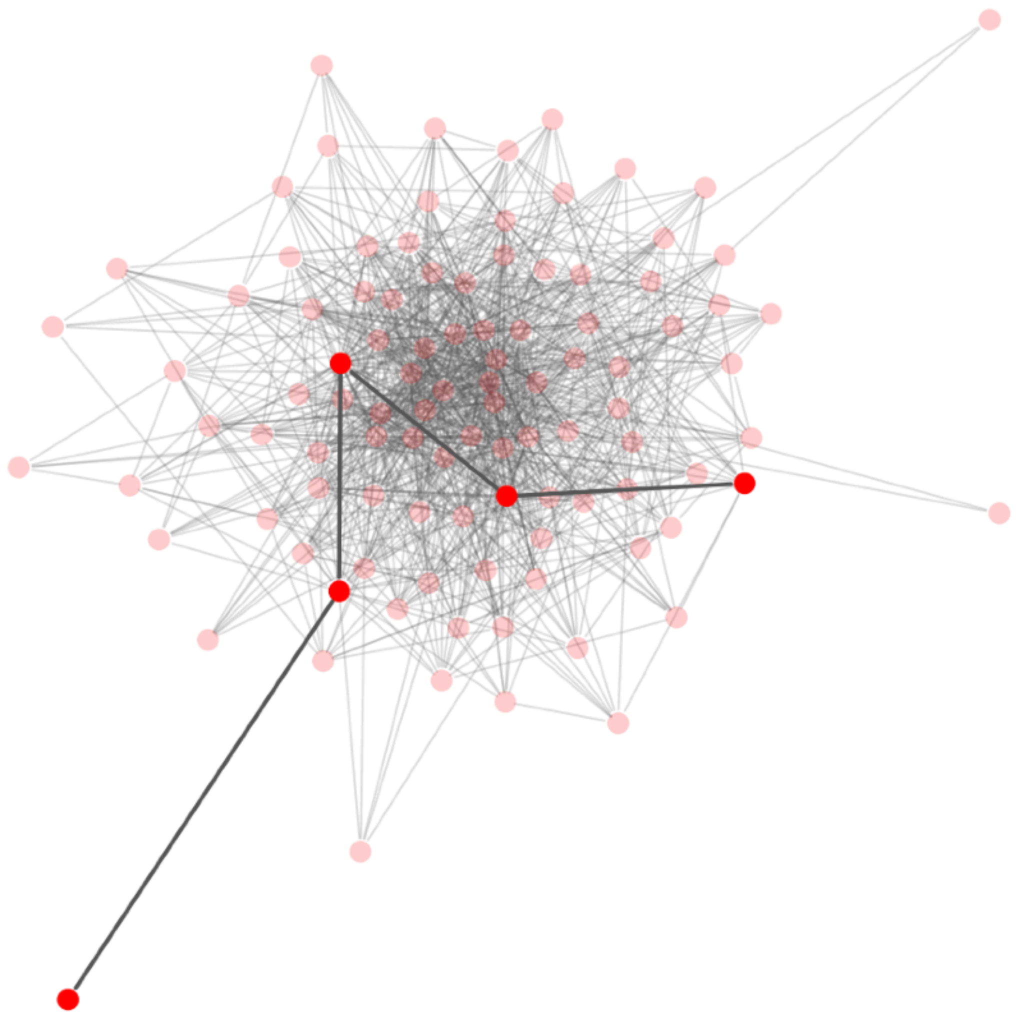

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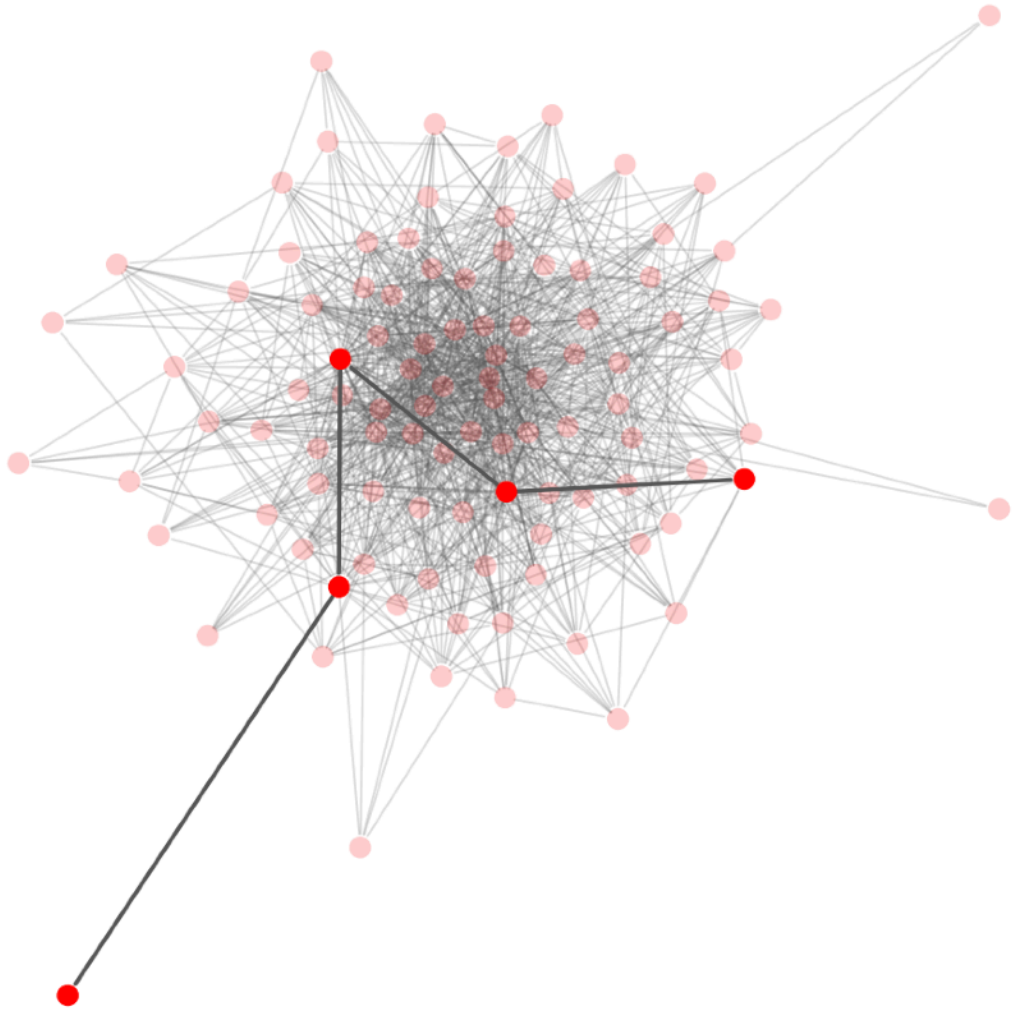

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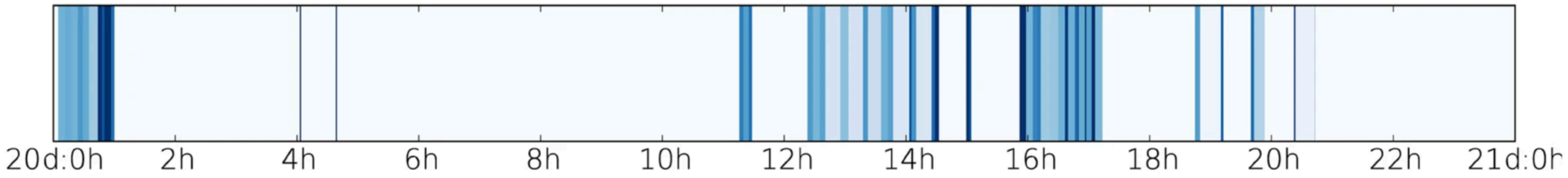

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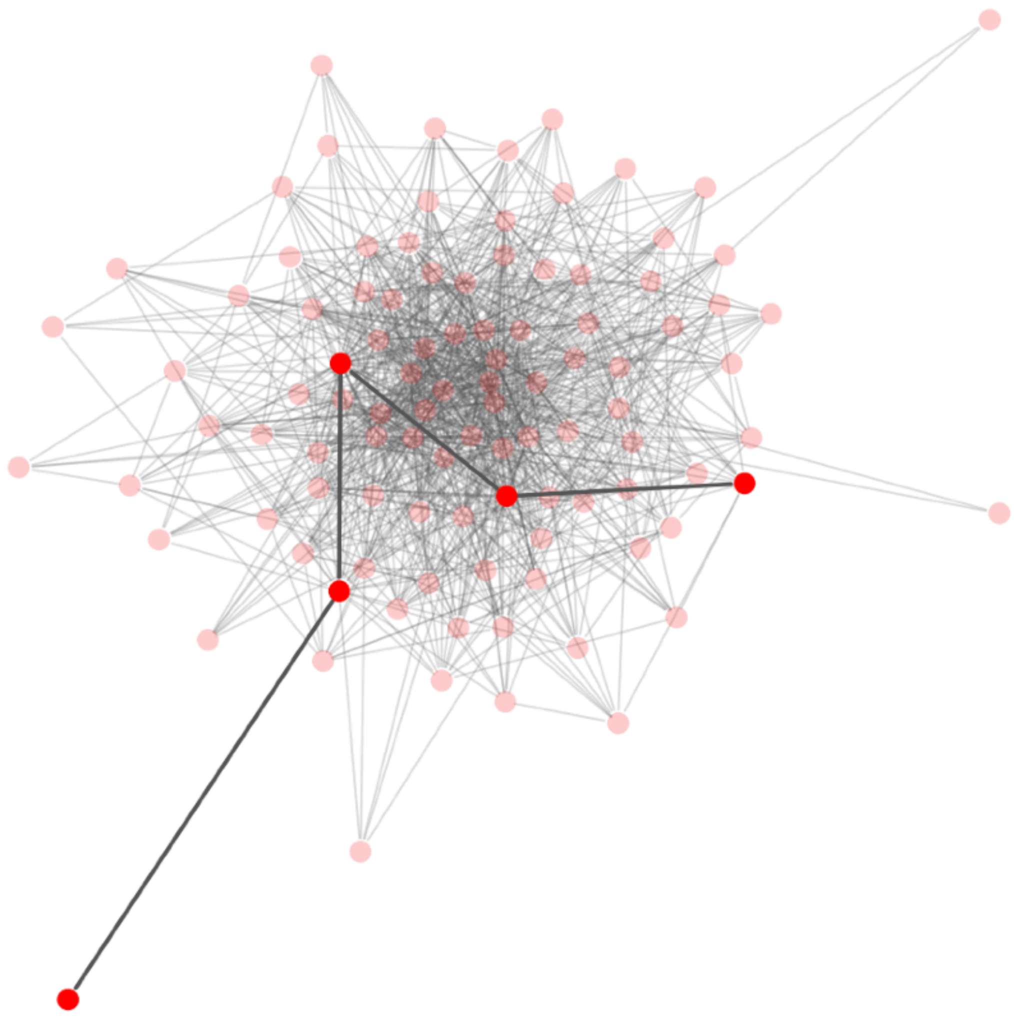

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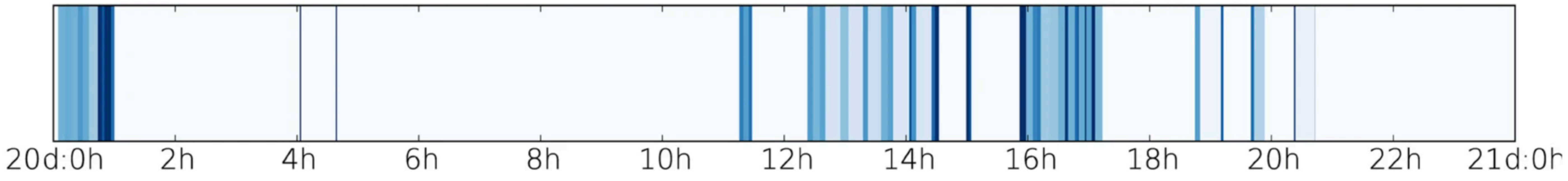

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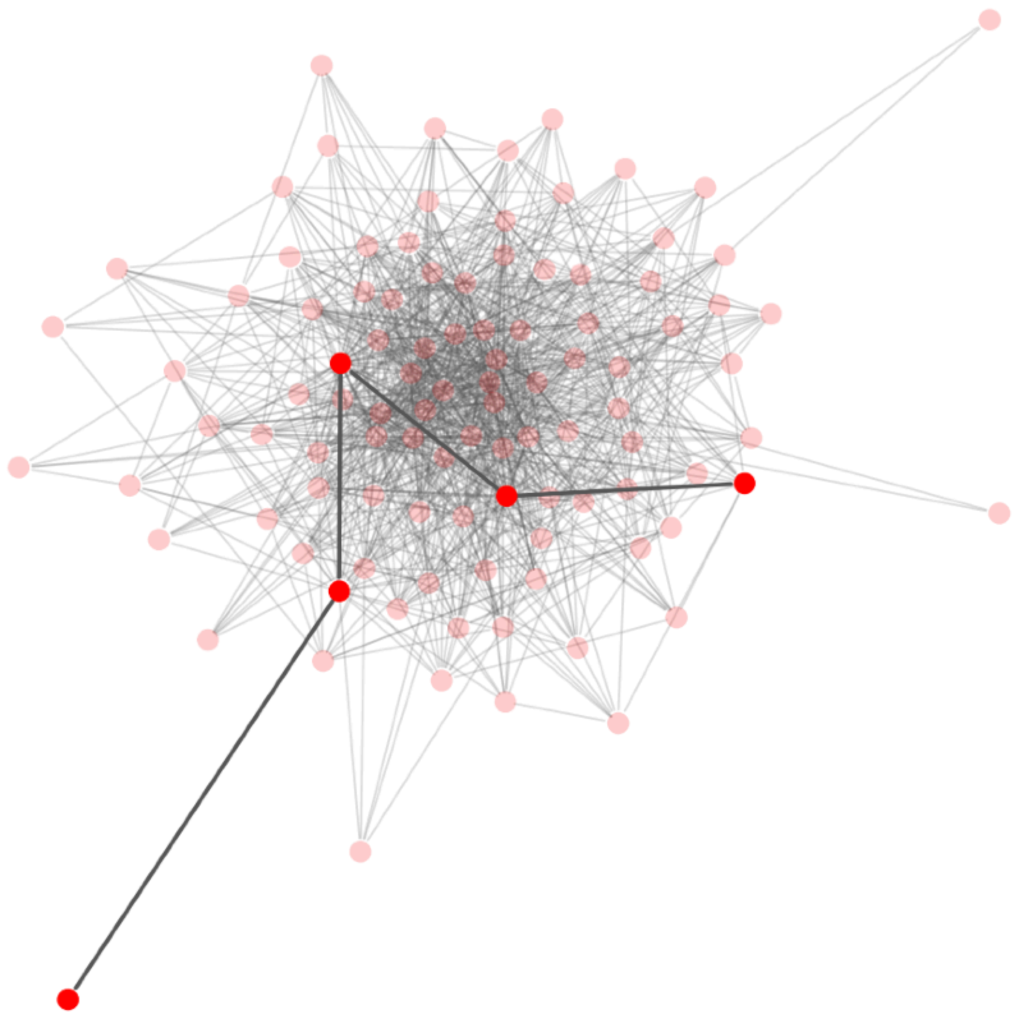

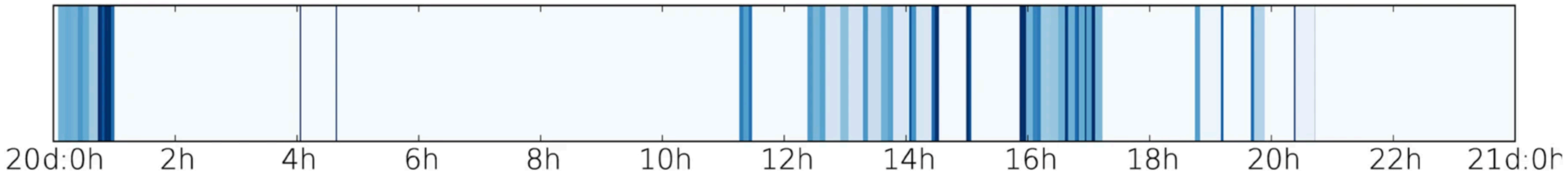

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Systematic methods to characterize simultaneously the temporal and topological relations of contacts/events are still missing.

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Do contacts close in time occur close in topology too?

Systematic characterization of contacts

Systematic characterization of contacts

- Interrelation of topological and temporal distance of contacts

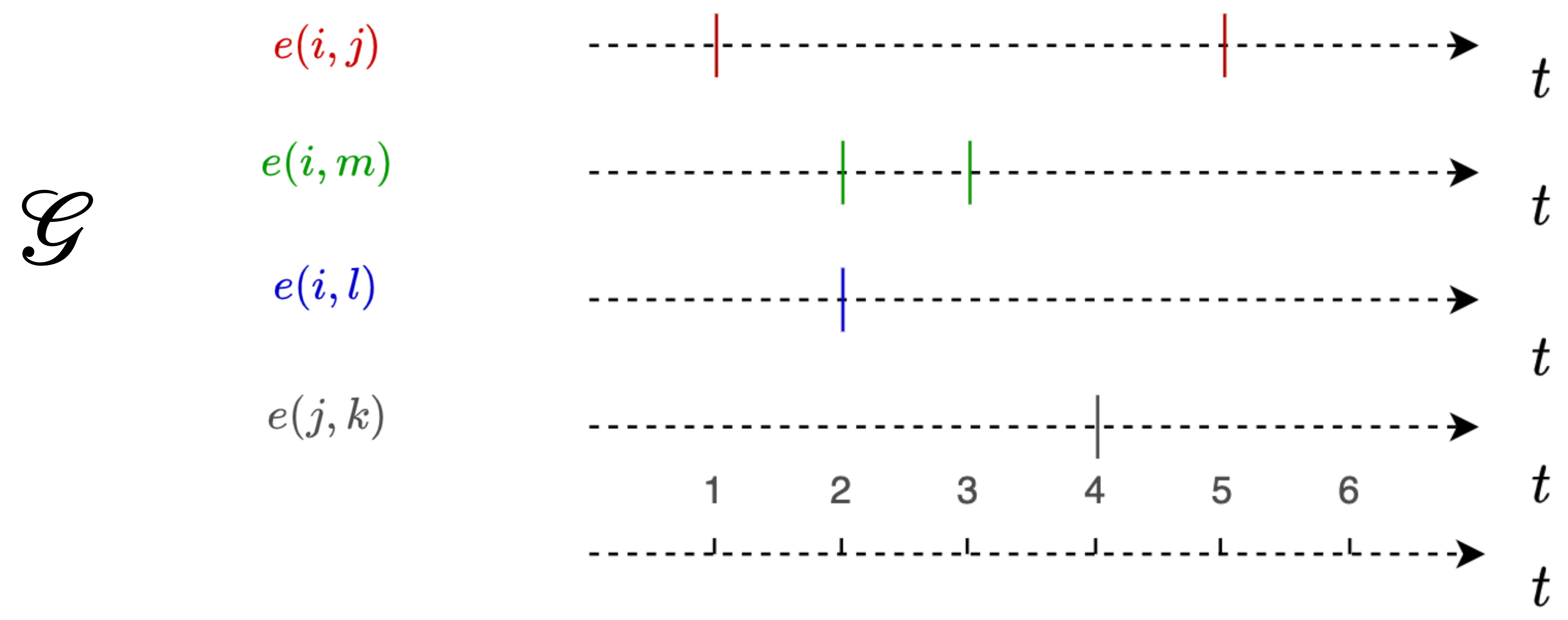
Systematic characterization of contacts

- Interrelation of topological and temporal distance of contacts
- Local analysis of temporal correlation around a link

Contacts of which evolving networks?

Network	N	$\|\mathcal{L}_W\|$	$\|\mathcal{S}\|$	$\|\mathcal{C}\|$	T	dt	*contact type*
DNC Mail Part 2 (DNC_2*)	1598	4085	17300	30091	2861358	1	*virtual*
Manufacturing Email (ME*)	167	3250	57791	82281	23430482	1	*virtual*
College Messages (CM*)	1892	13833	58905	59789	16362751	1	*virtual*
Email EU (EEU*)	986	16025	206311	324933	44719809	1	*virtual*
Infectious (Infectious)	410	2765	1392	17298	1421	20	*physical*
Primary School (PS)	242	8317	3099	125771	3098	20	*physical*
High School 2012 (HS2012)	180	2220	11267	45047	14114	20	*physical*
High School 2013 (HS2013)	327	5818	7371	188504	7370	20	*physical*
Hypertext 2009 (HT2009)	113	2196	5243	20818	7226	20	*physical*
SFHH Conference (SFHH)	403	9565	3508	70261	3799	20	*physical*
Workplace 2013 (WP)	92	755	7095	9827	17844	20	*physical*
Workplace 2015 (WP2)	217	4274	18479	78246	20946	20	*physical*
Hospital (Hospital)	75	1139	9452	32424	16026	20	*physical*

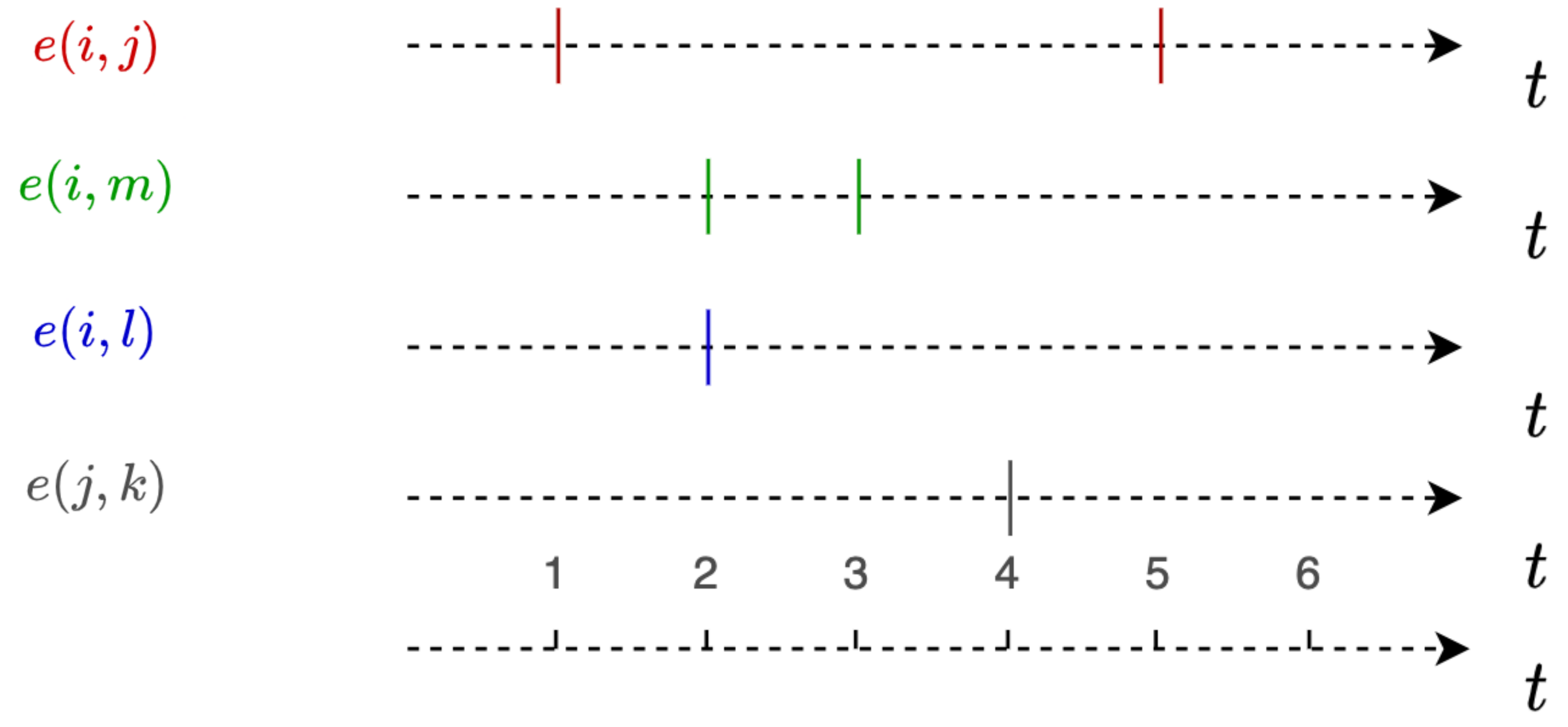
Randomised reference models



Gauvin, Laetitia, et al. "Randomized reference models for temporal networks." *arXiv preprint arXiv:1806.04032* (2018).

Randomised reference models

$\mathcal{G}$



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$e(i,m)$	2	4
$e(i,l)$	2	3
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reshuffle timestamps

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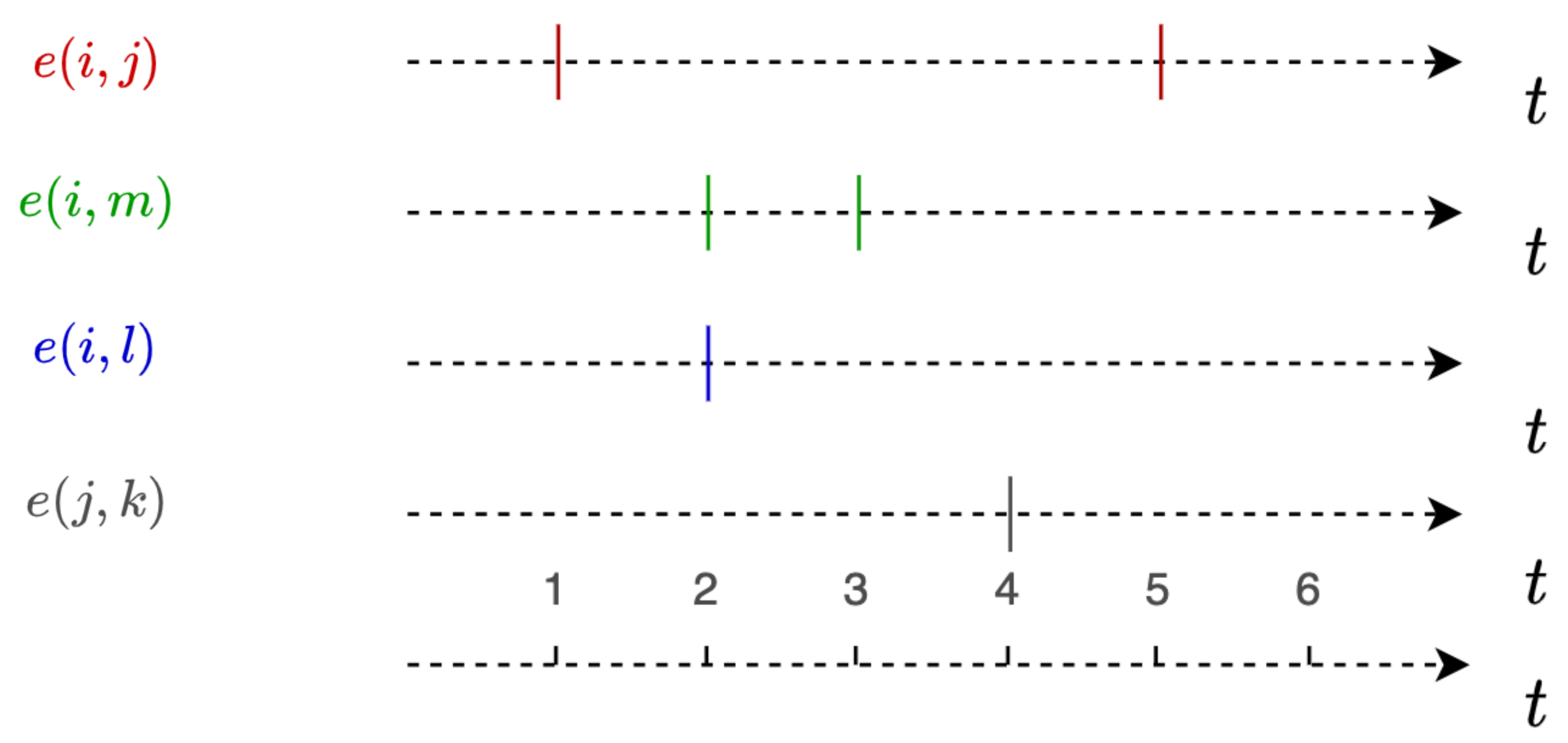

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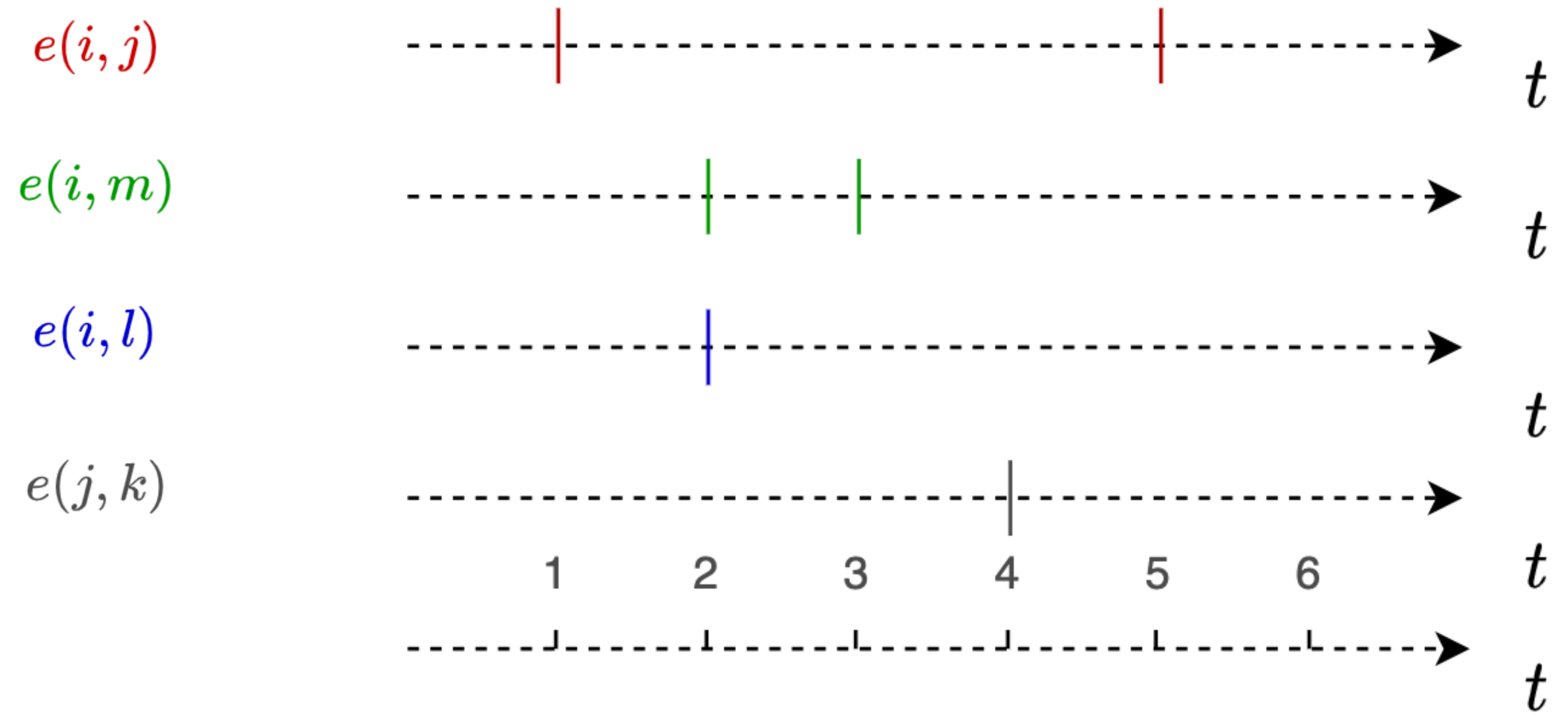

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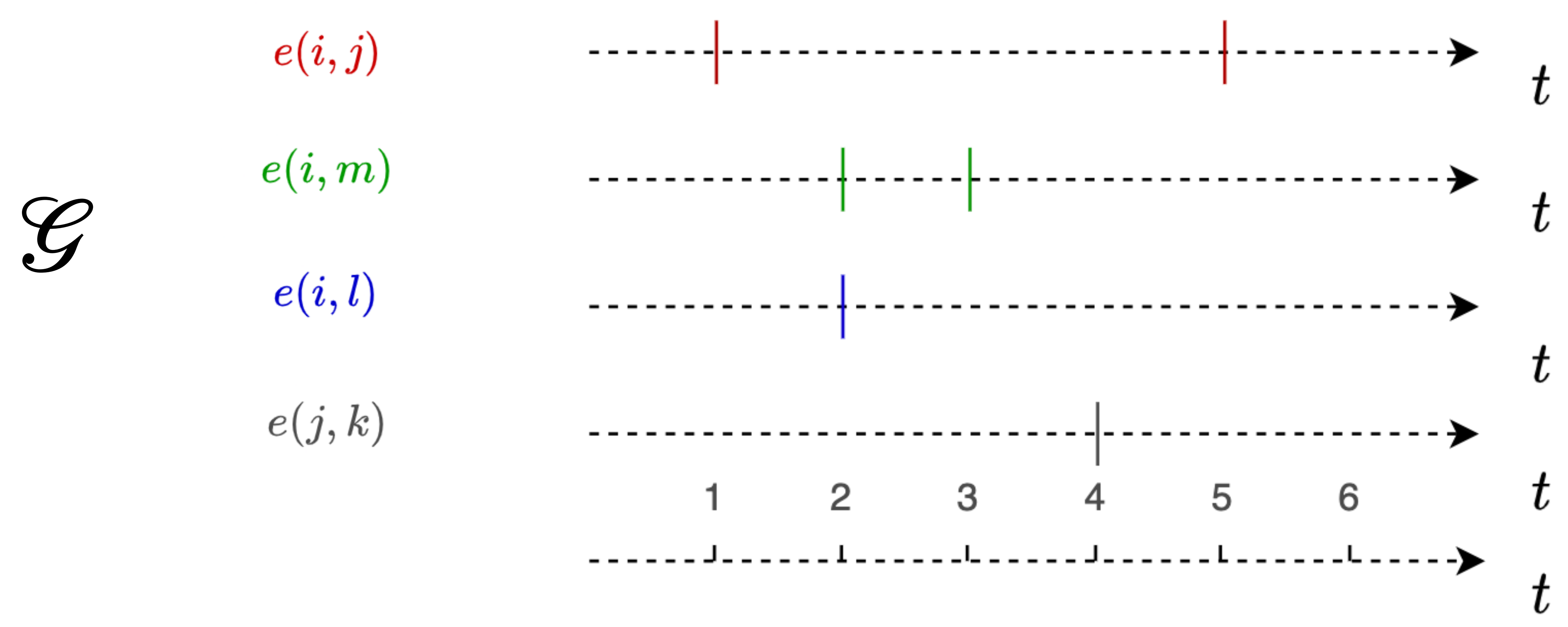

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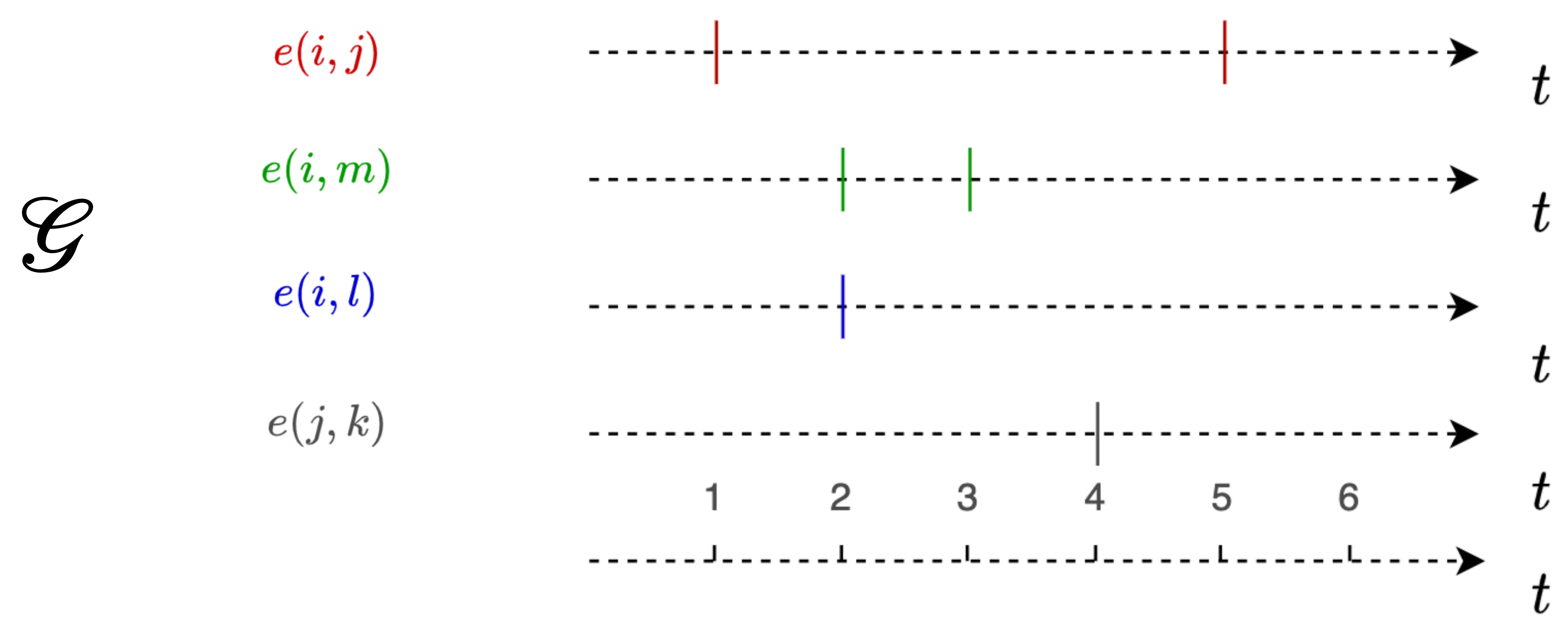

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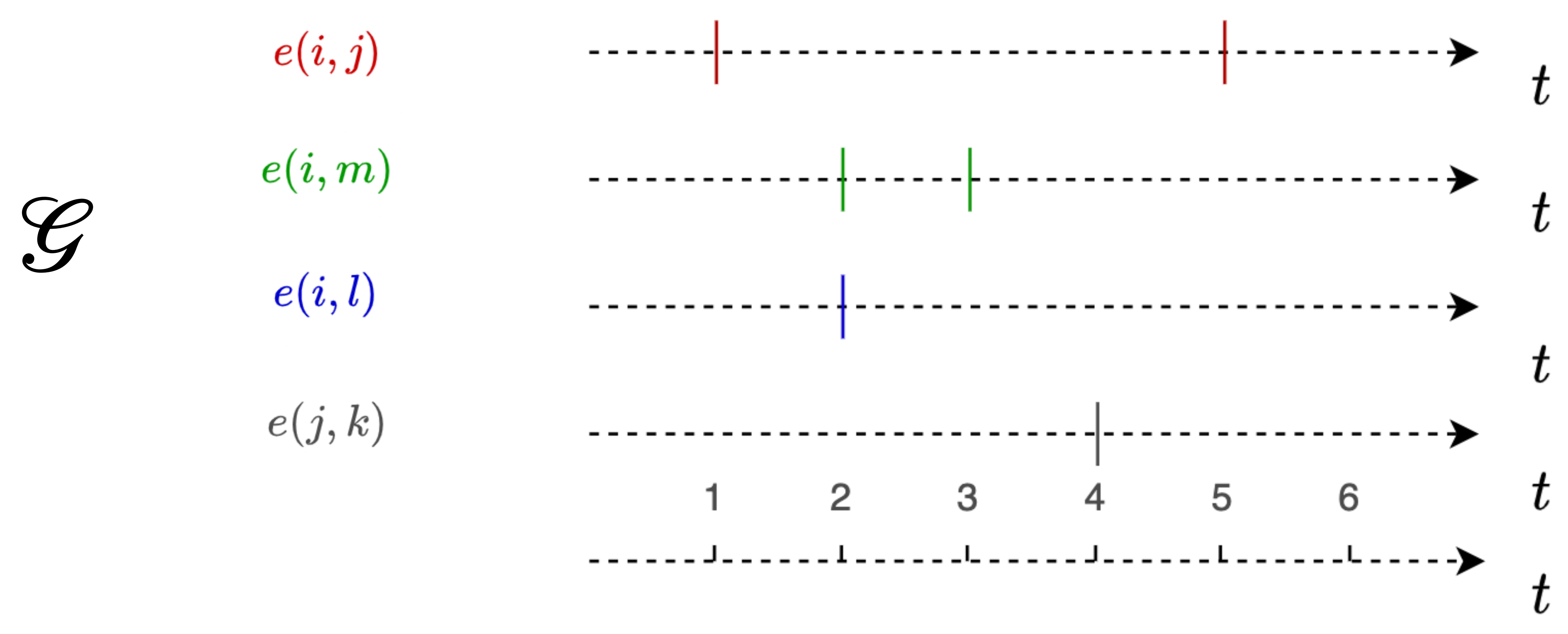

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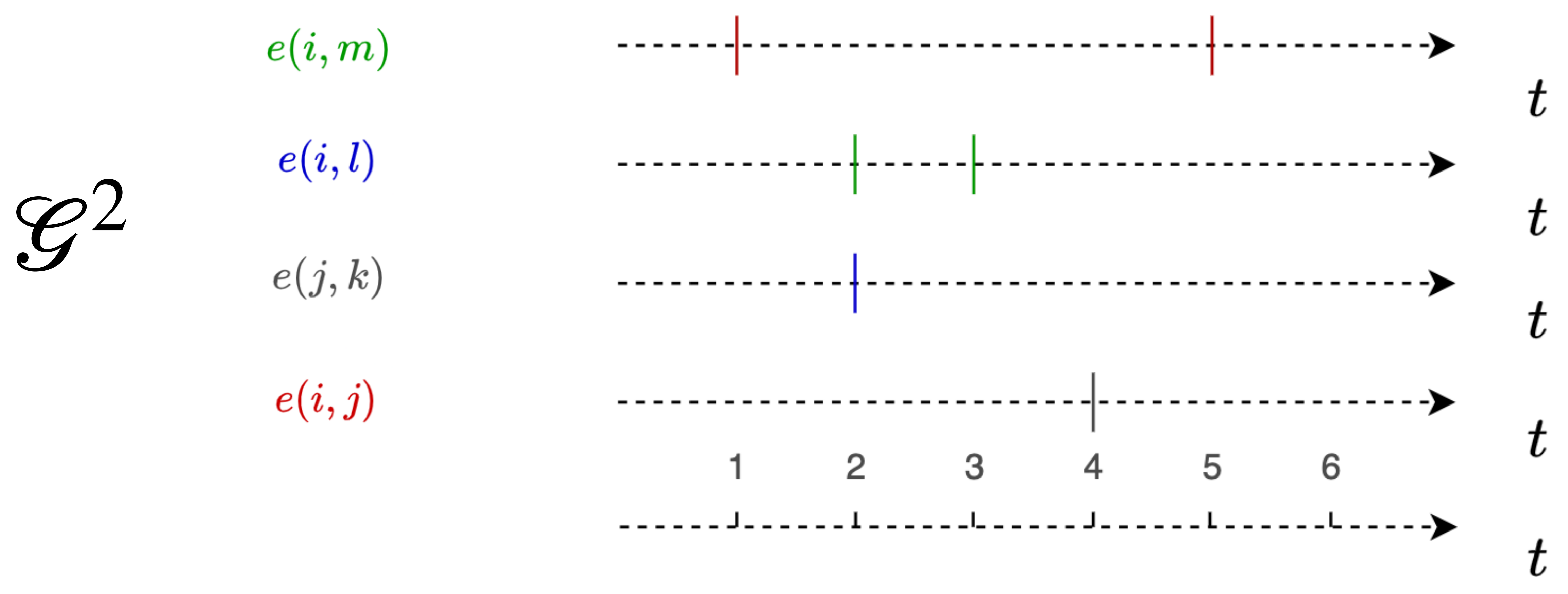

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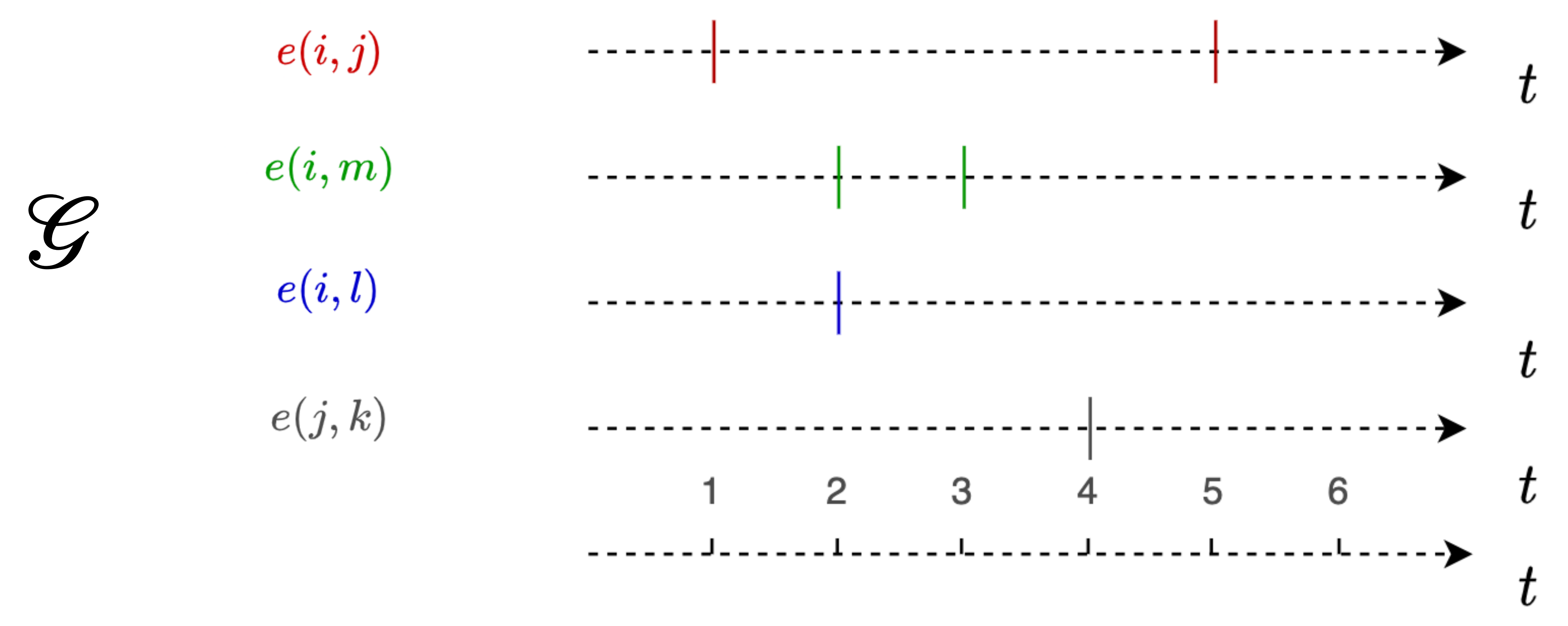

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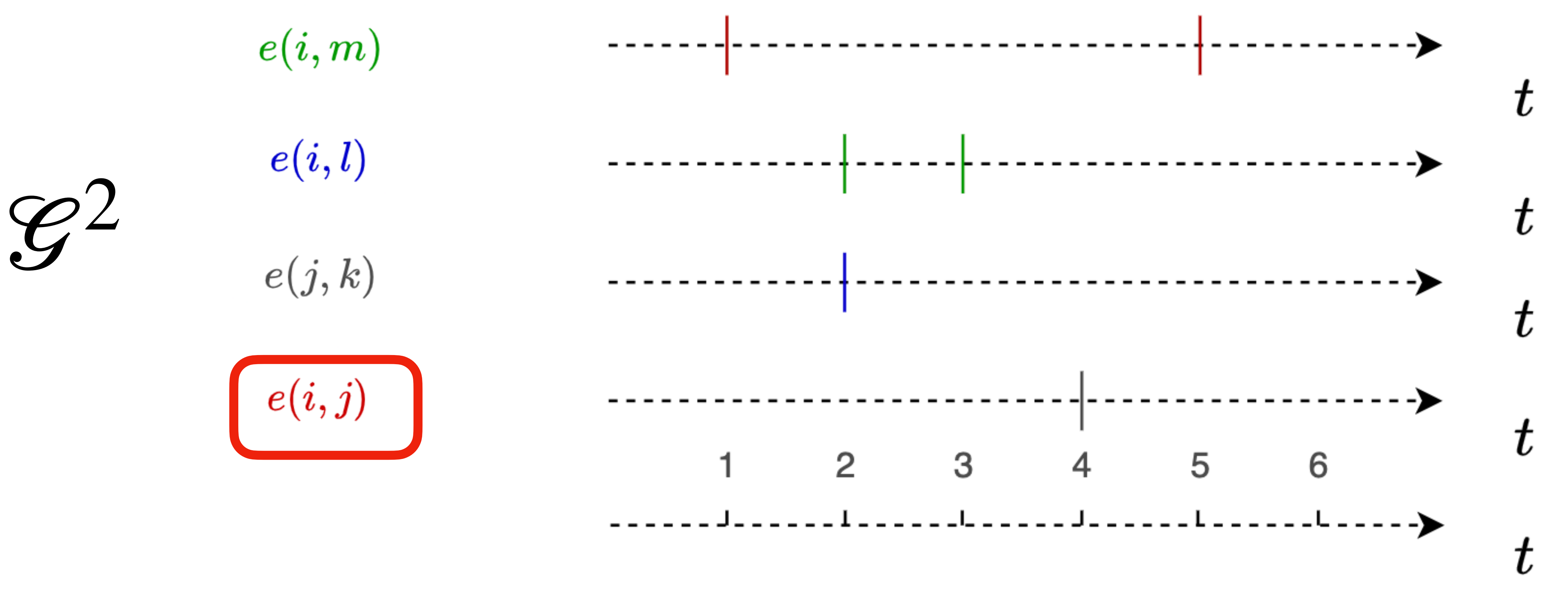

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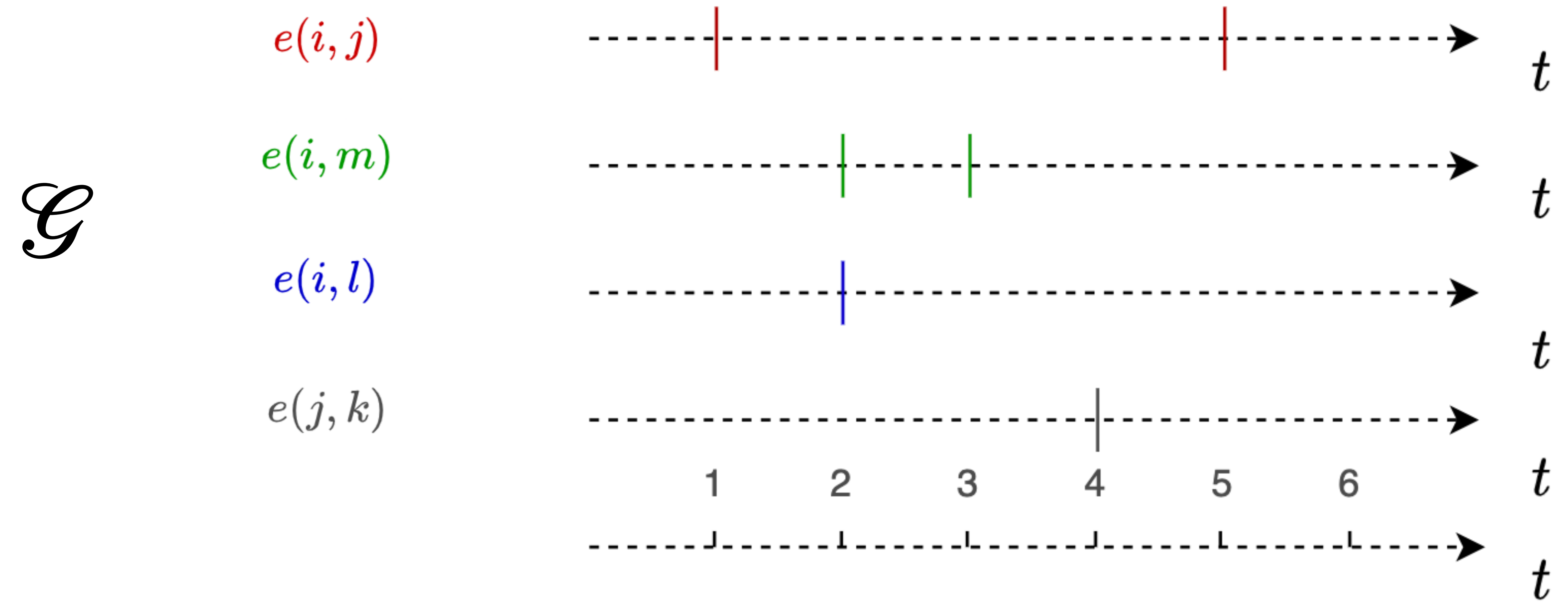

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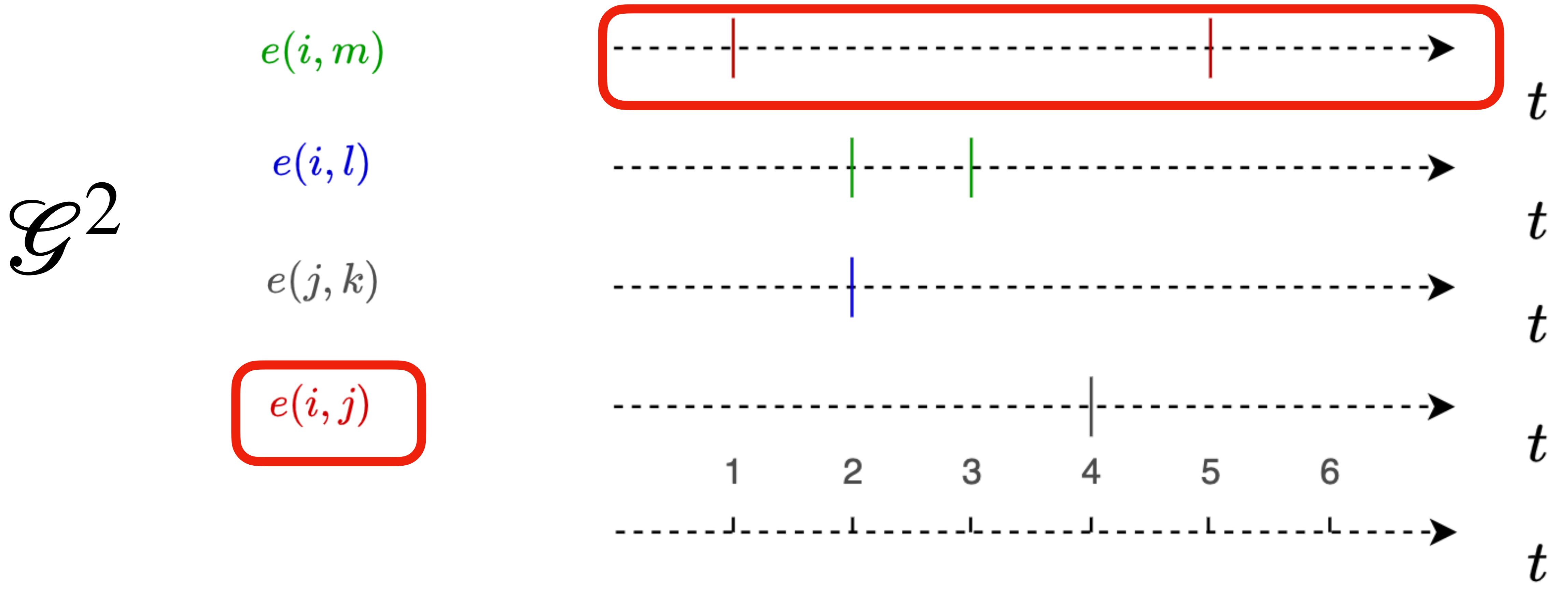

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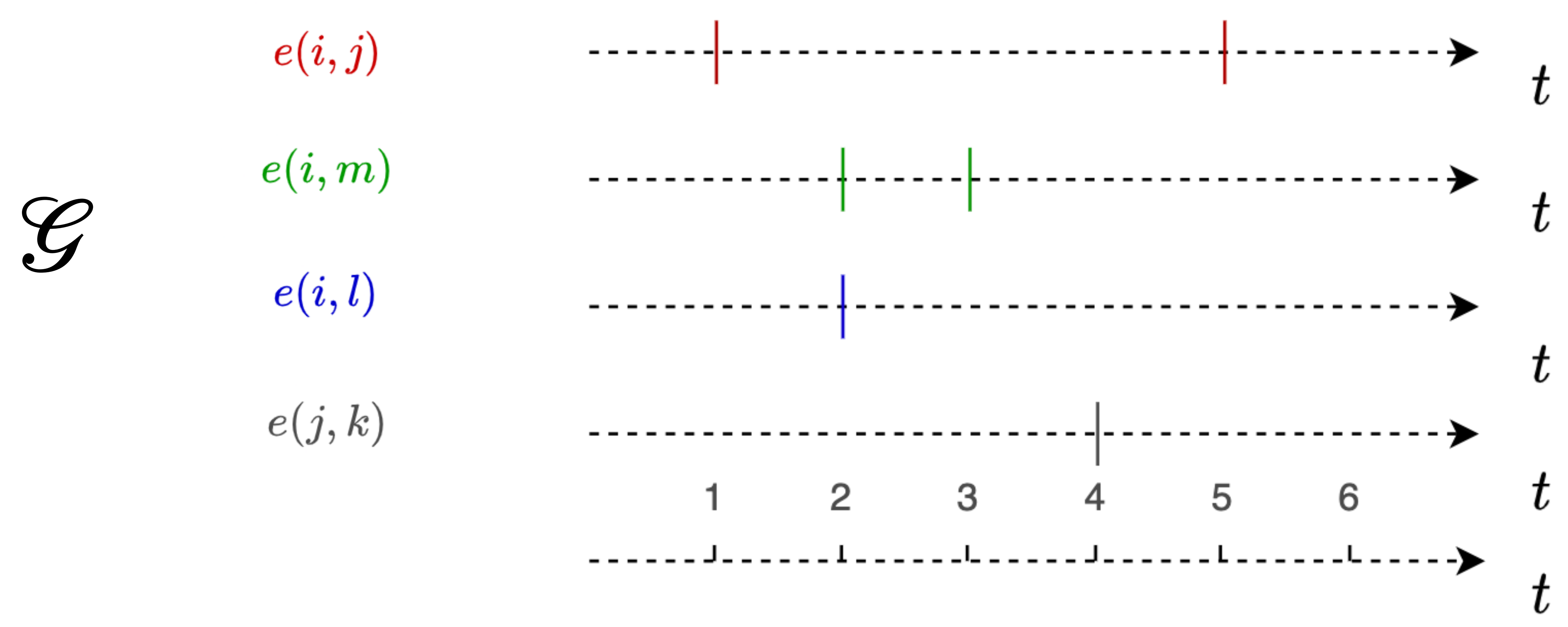

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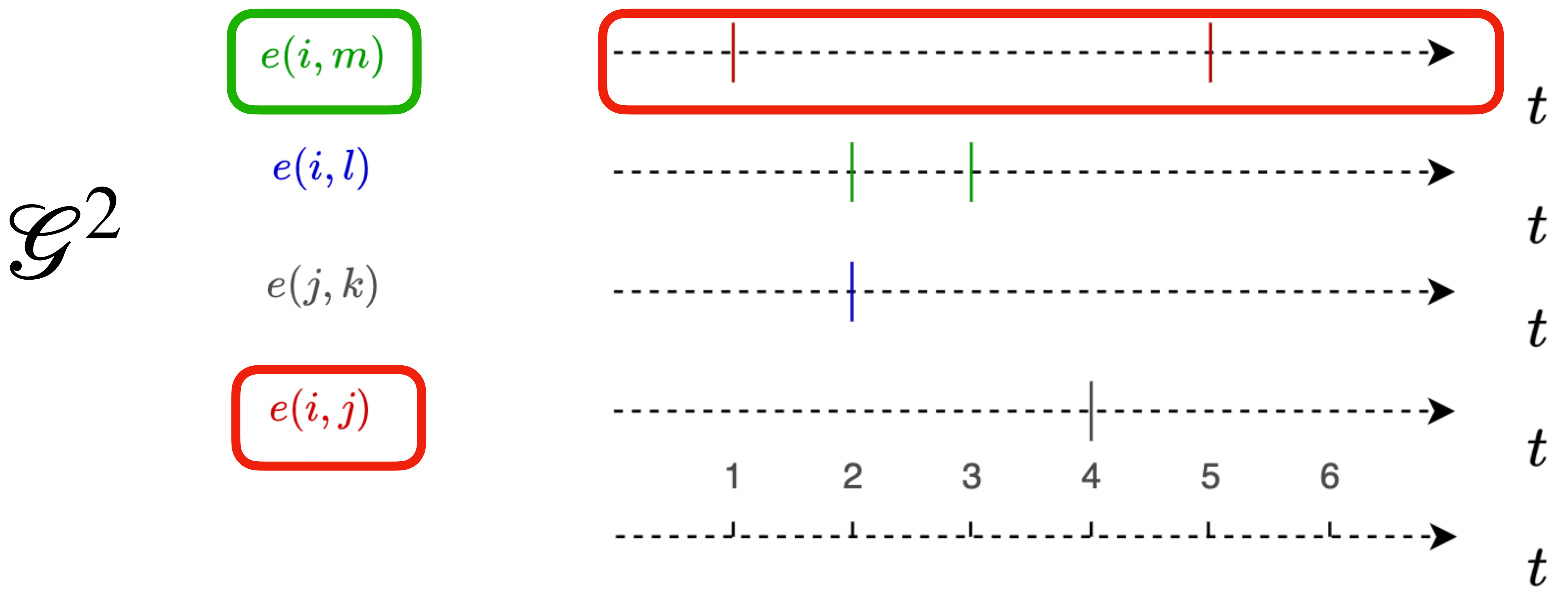

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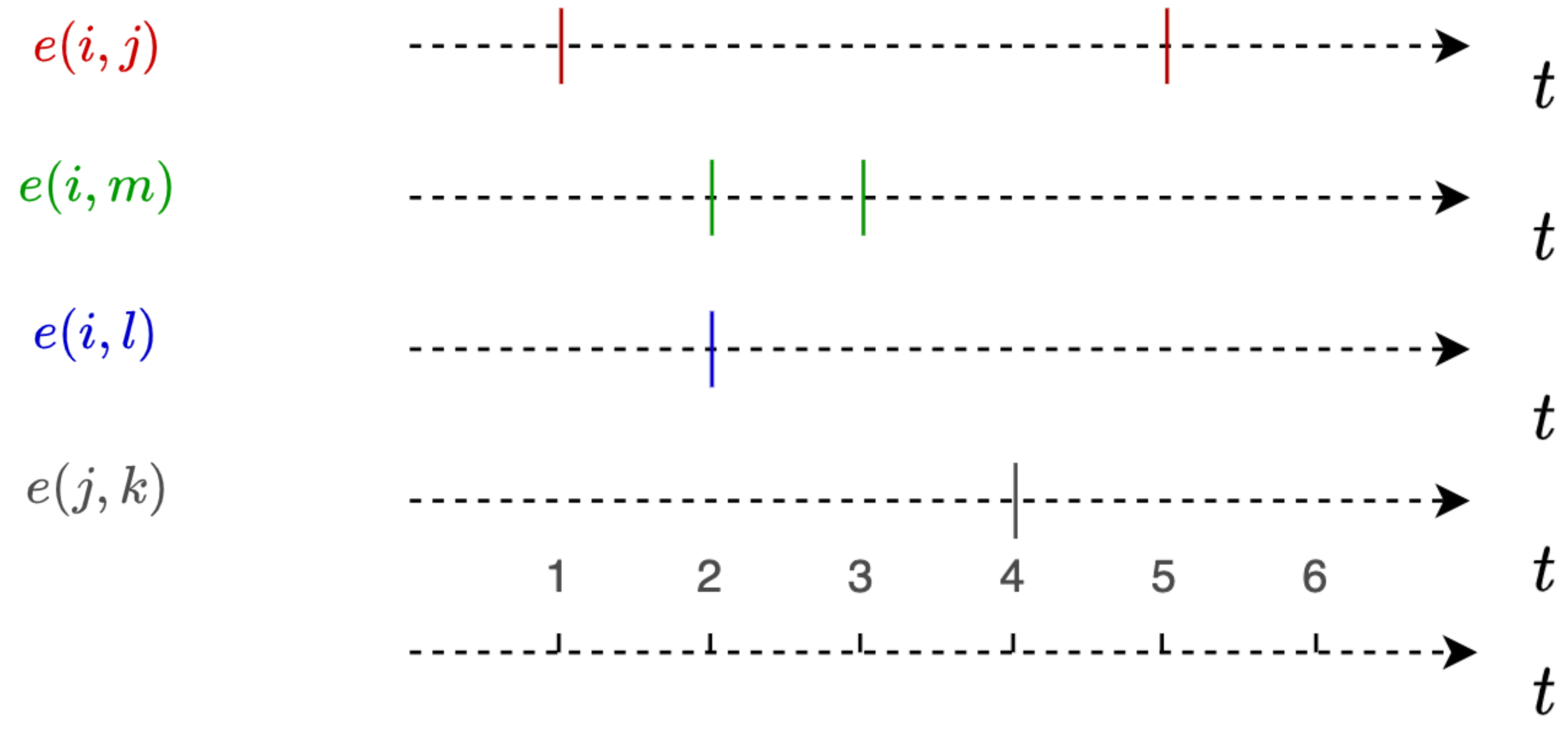

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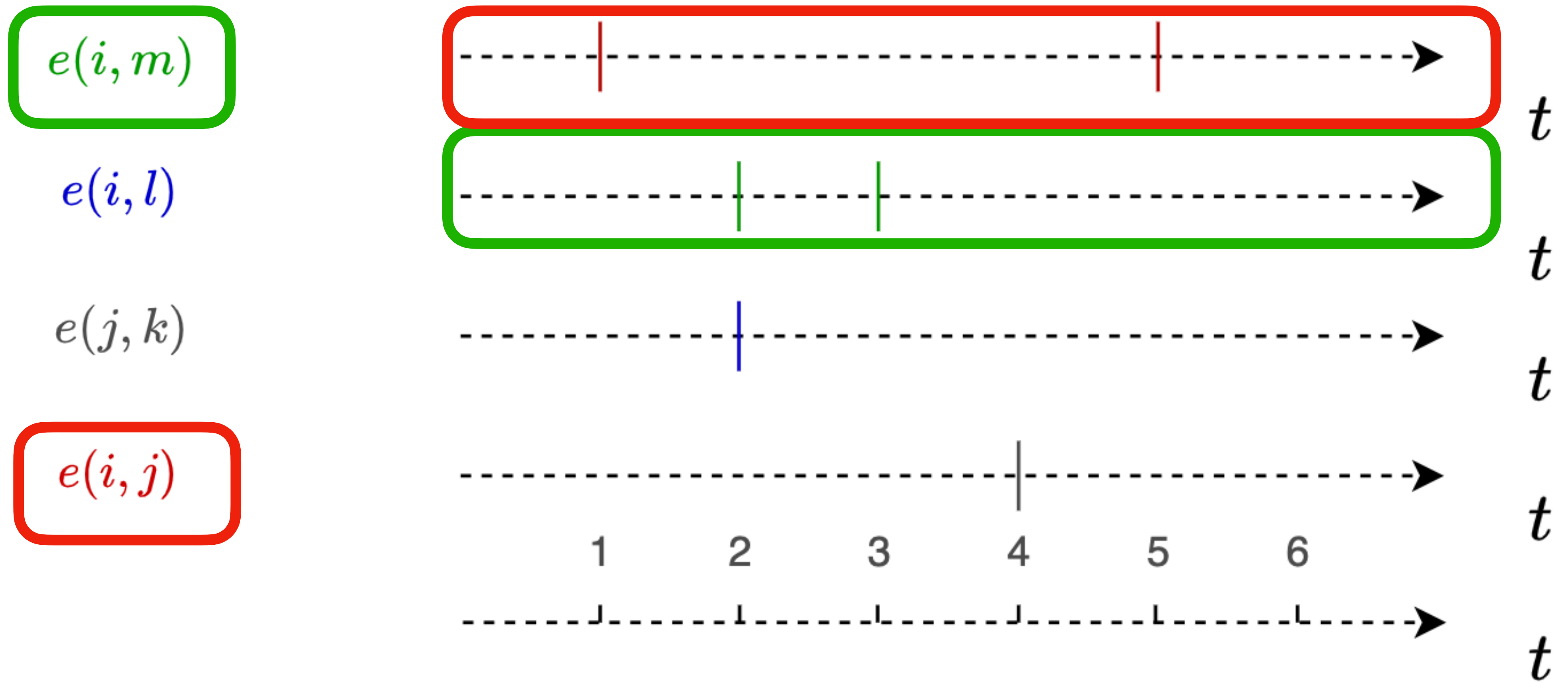

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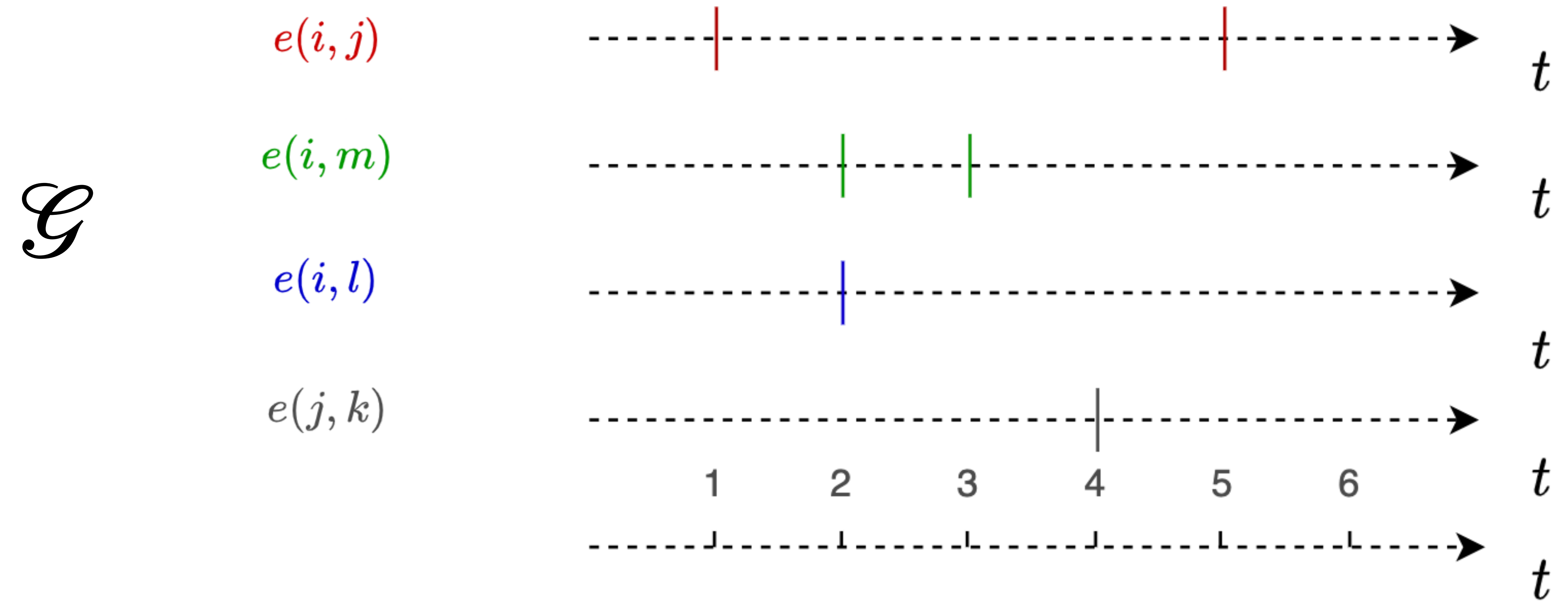

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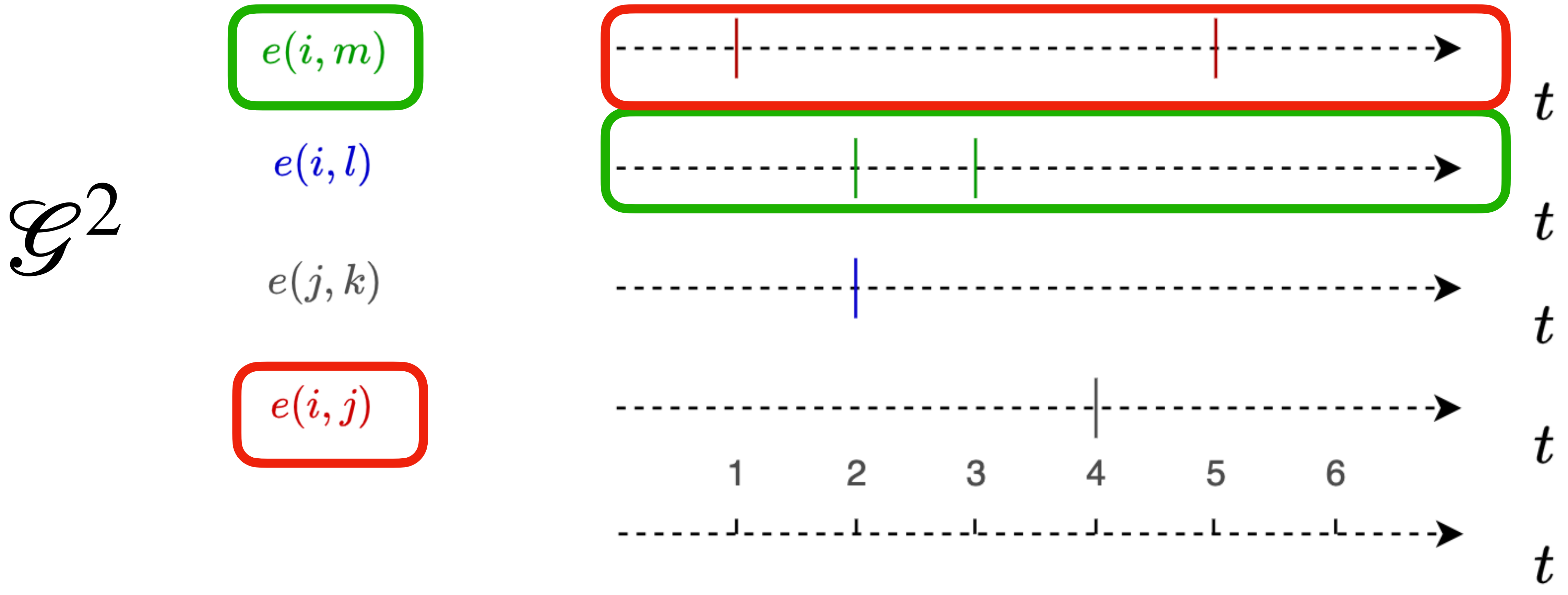

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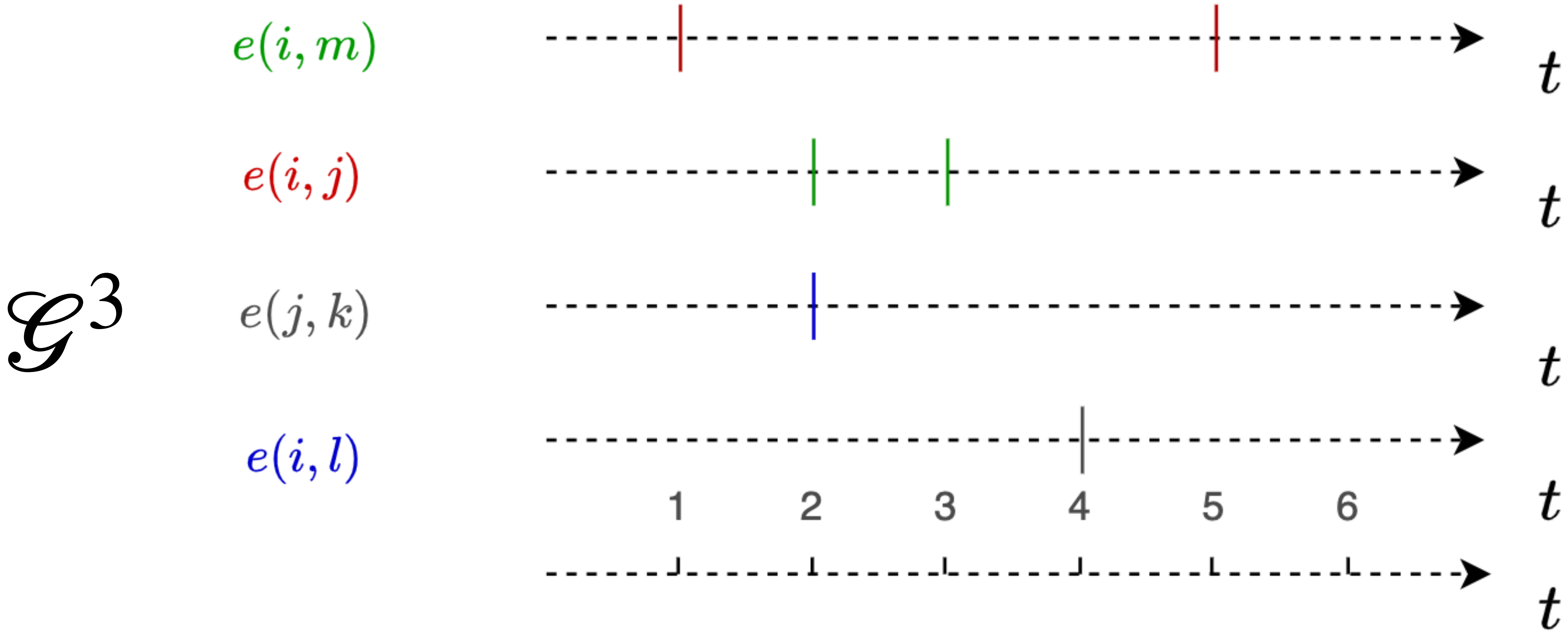

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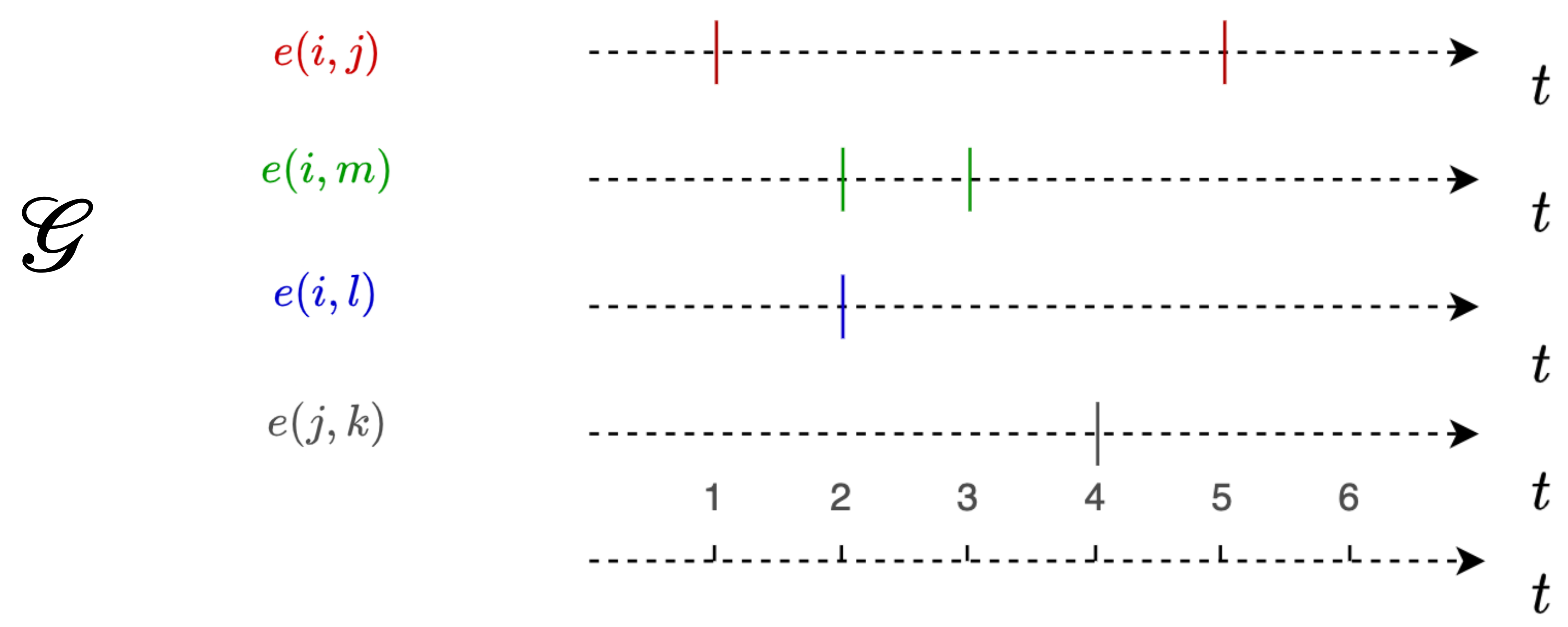

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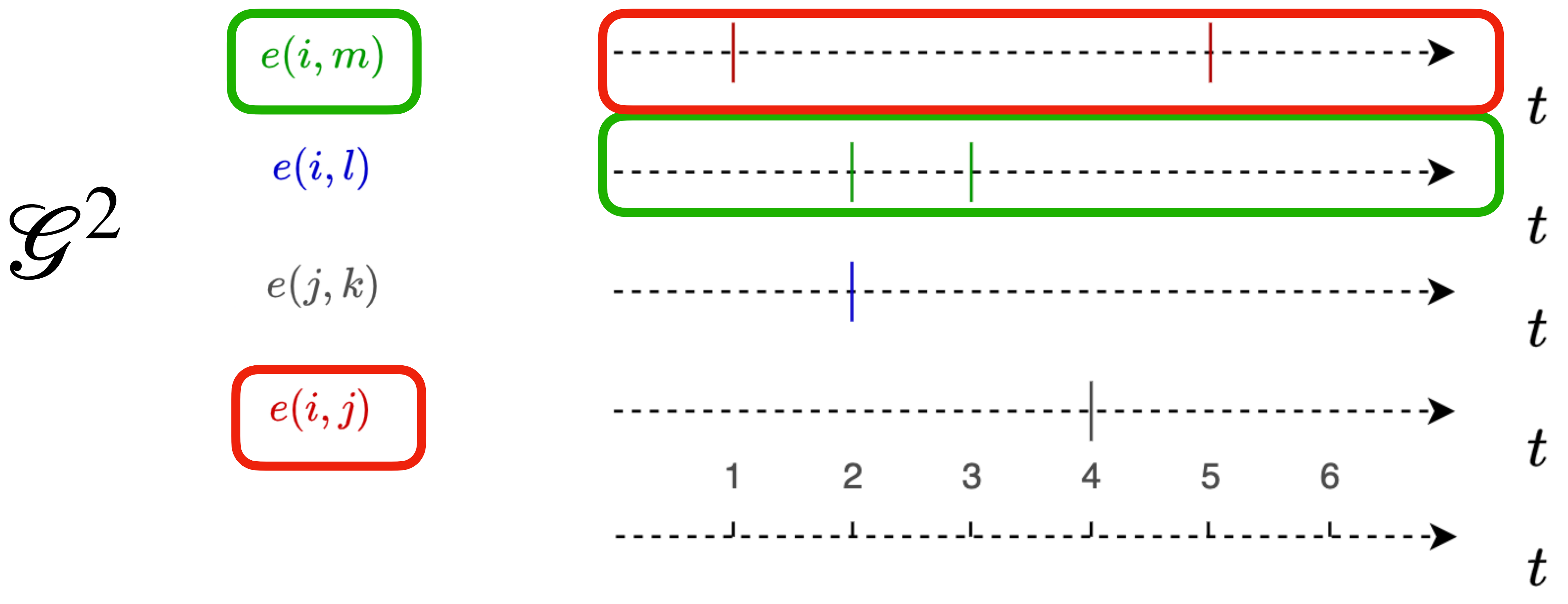

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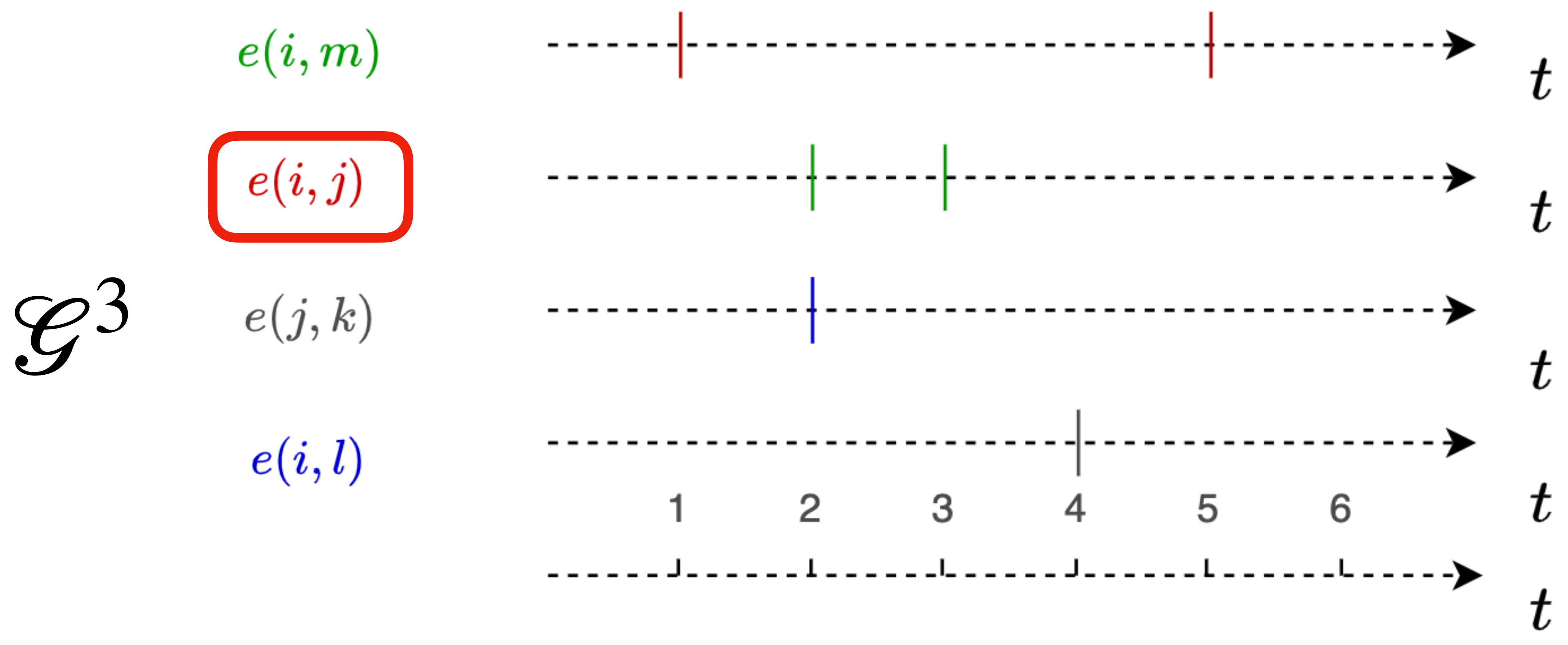

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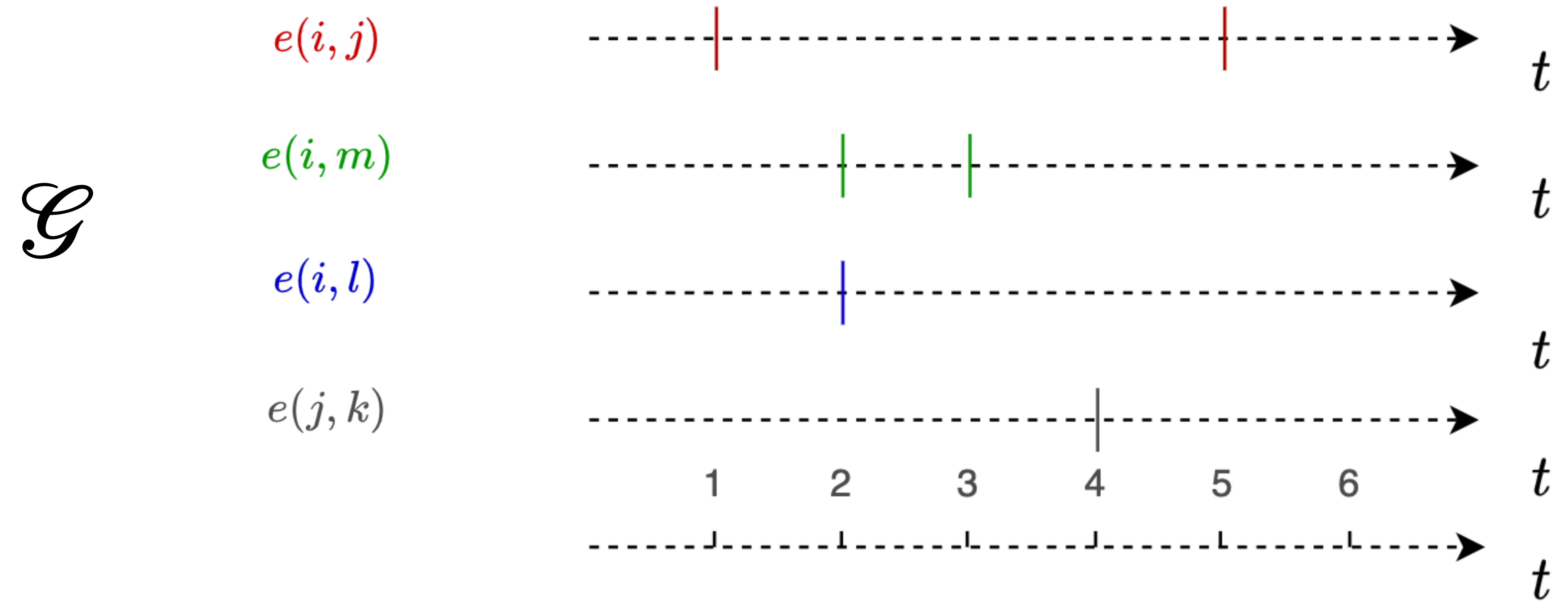

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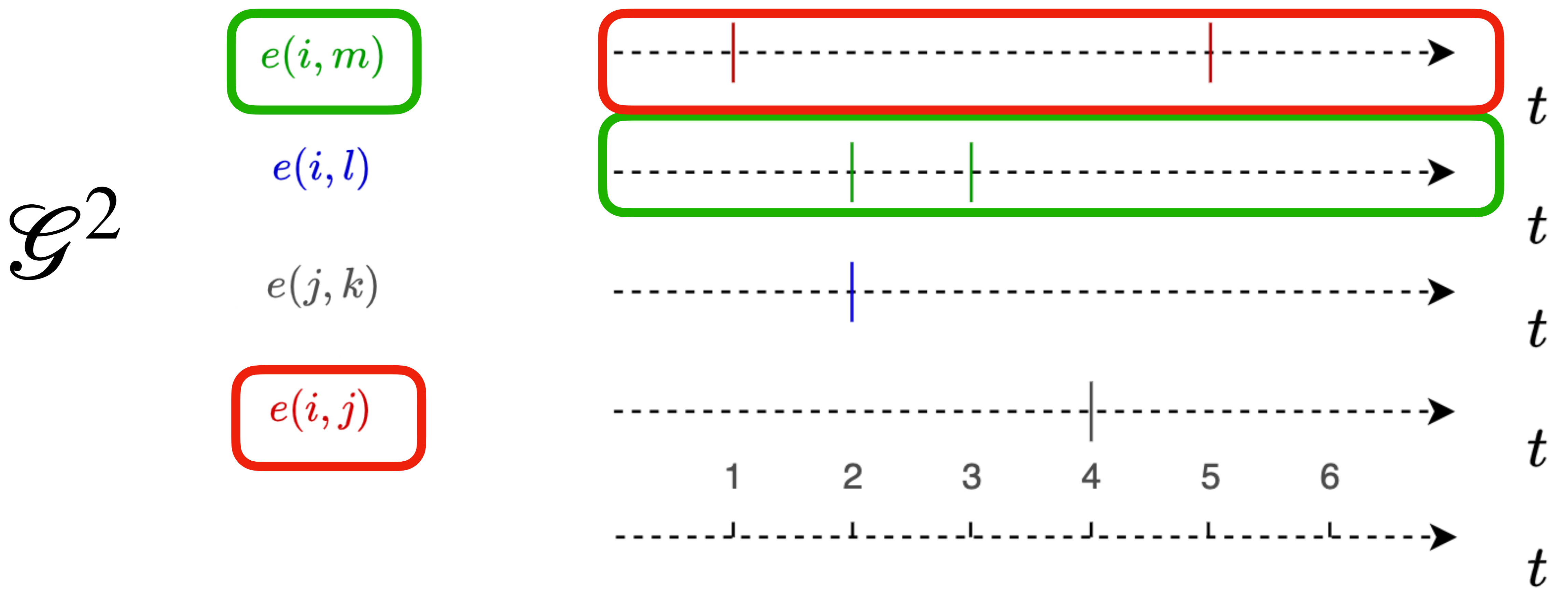

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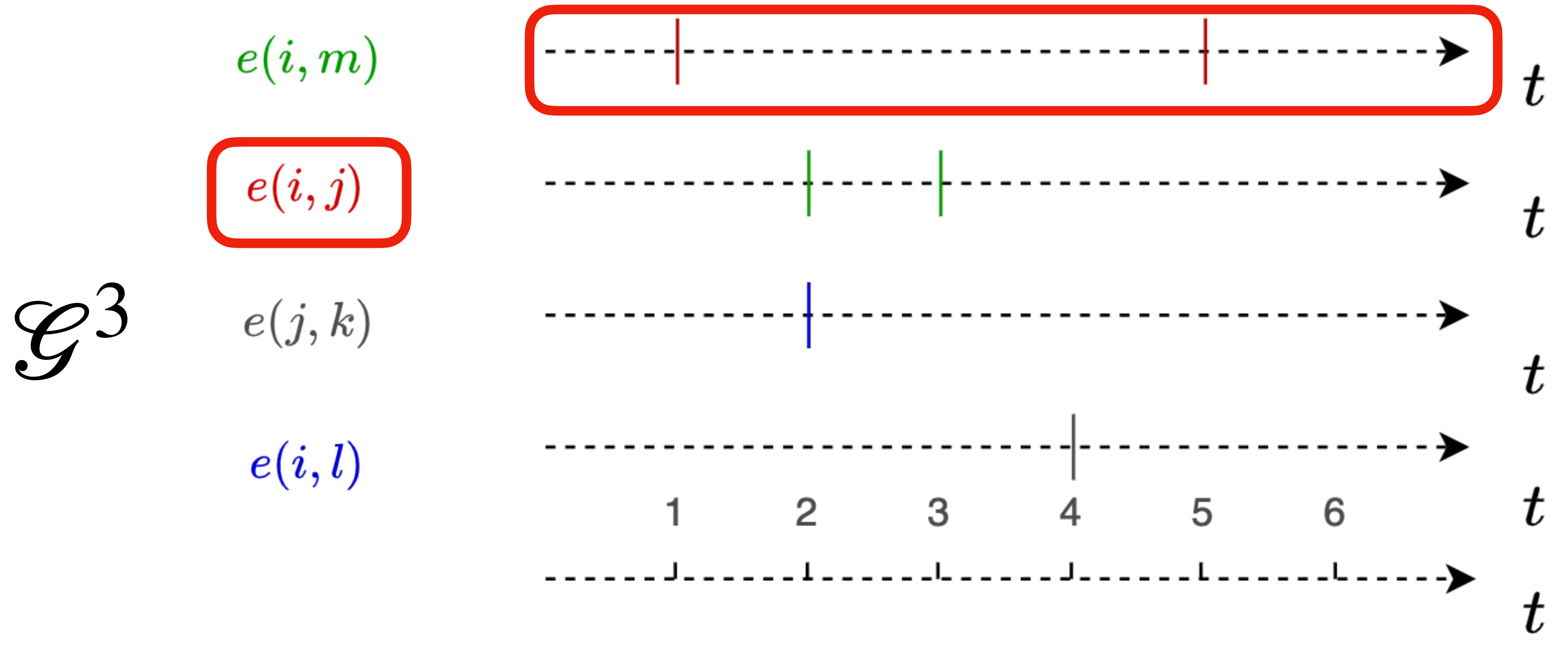

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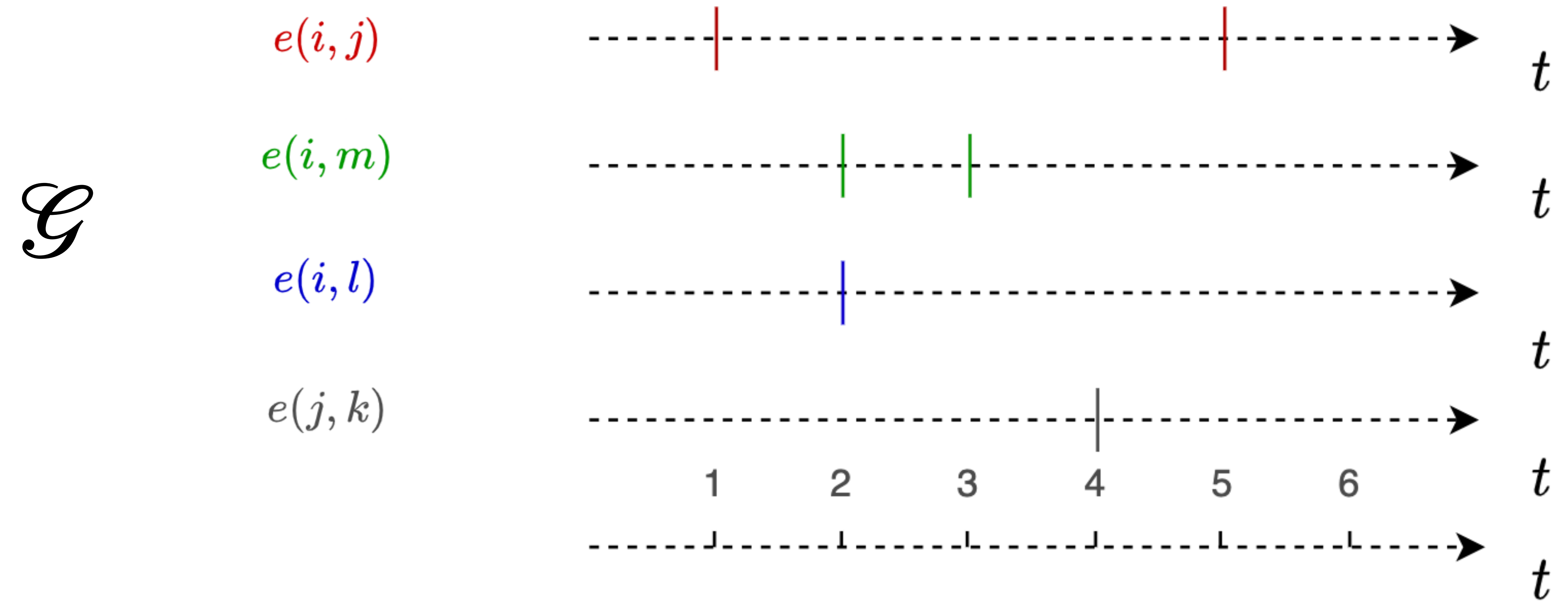

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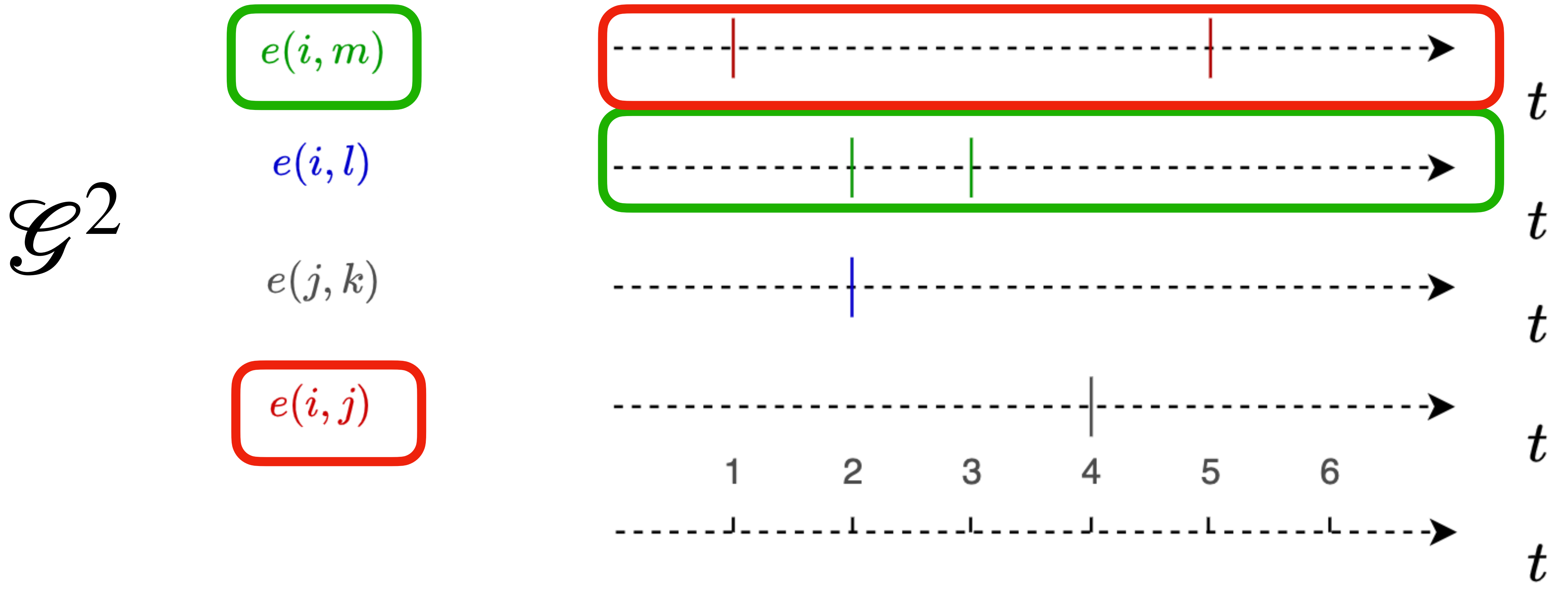
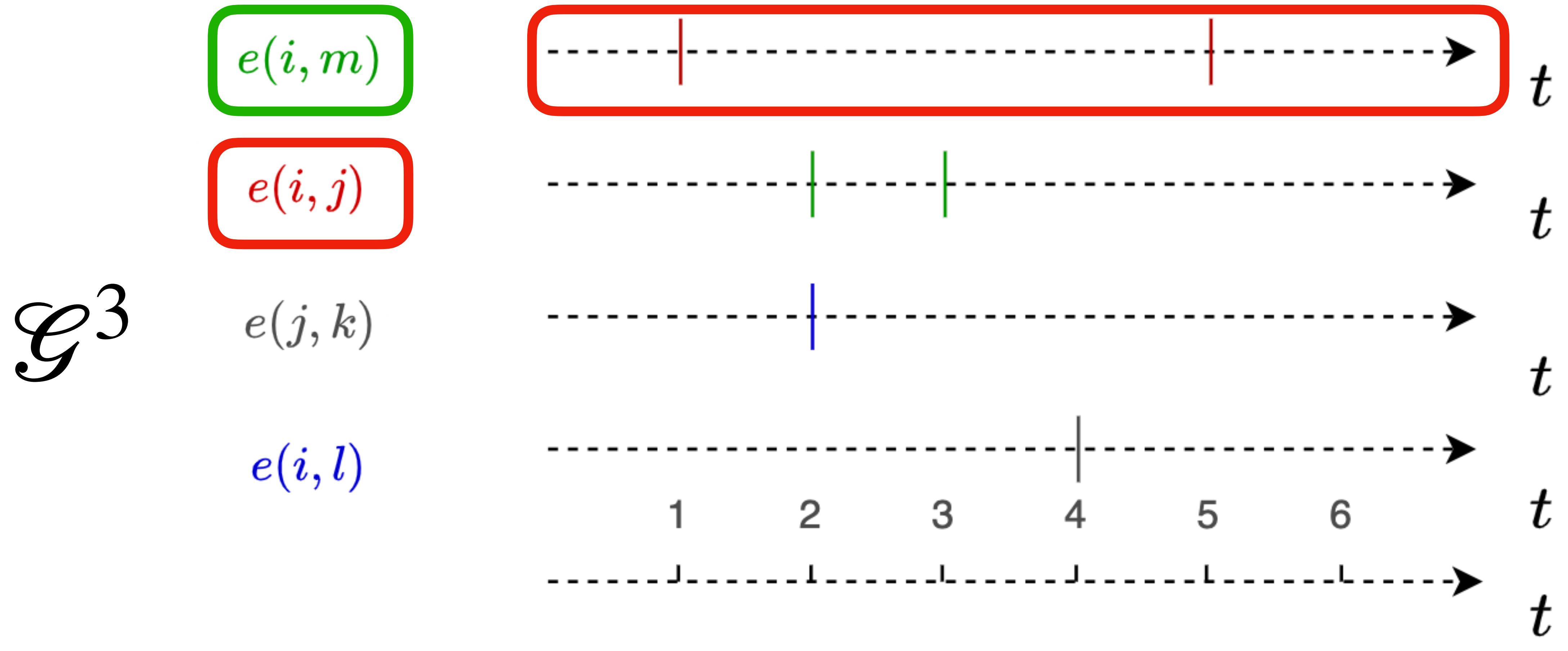
$e(i,l)$	2	3
$e(i,m)$	3	1
$e(j,k)$	4	2
$e(i,j)$	5	2

reshuffle timestamps



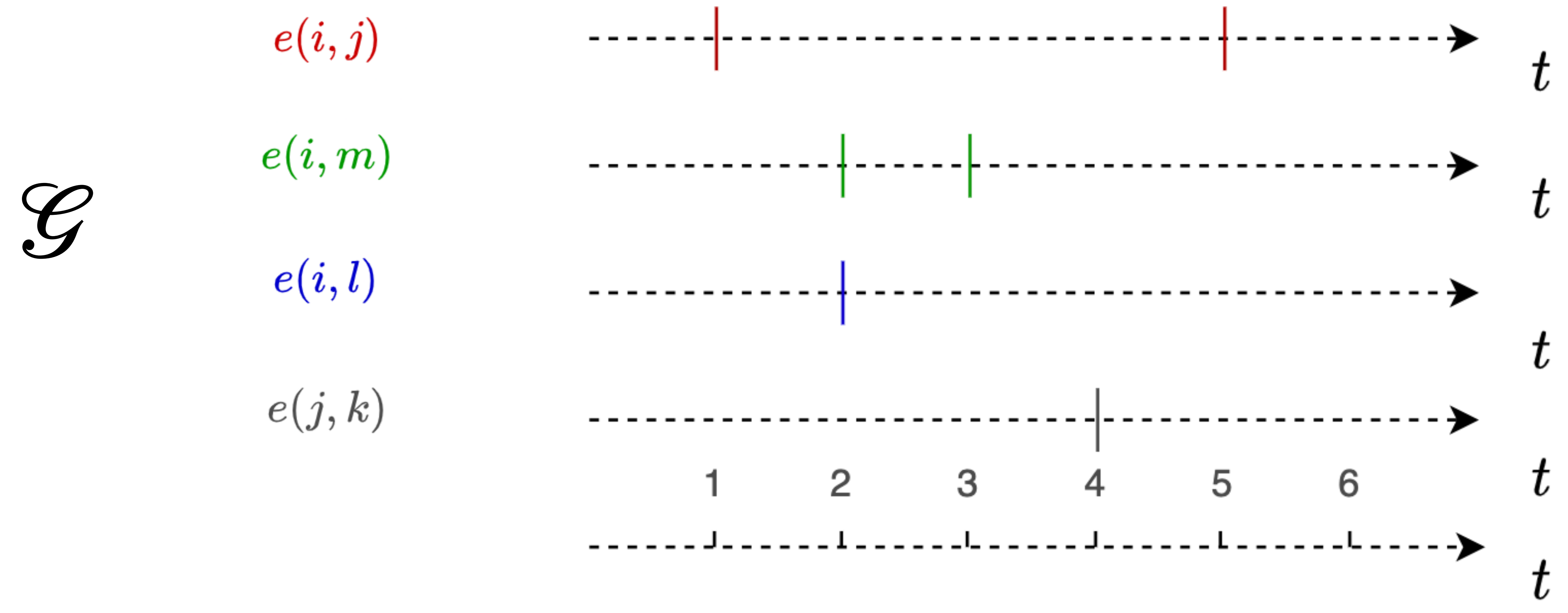
swap time series of links



swap time series of links with same number of activations

Gauvin, Laetitia, et al. "Randomized reference models for temporal networks." *arXiv preprint arXiv:1806.04032* (2018).

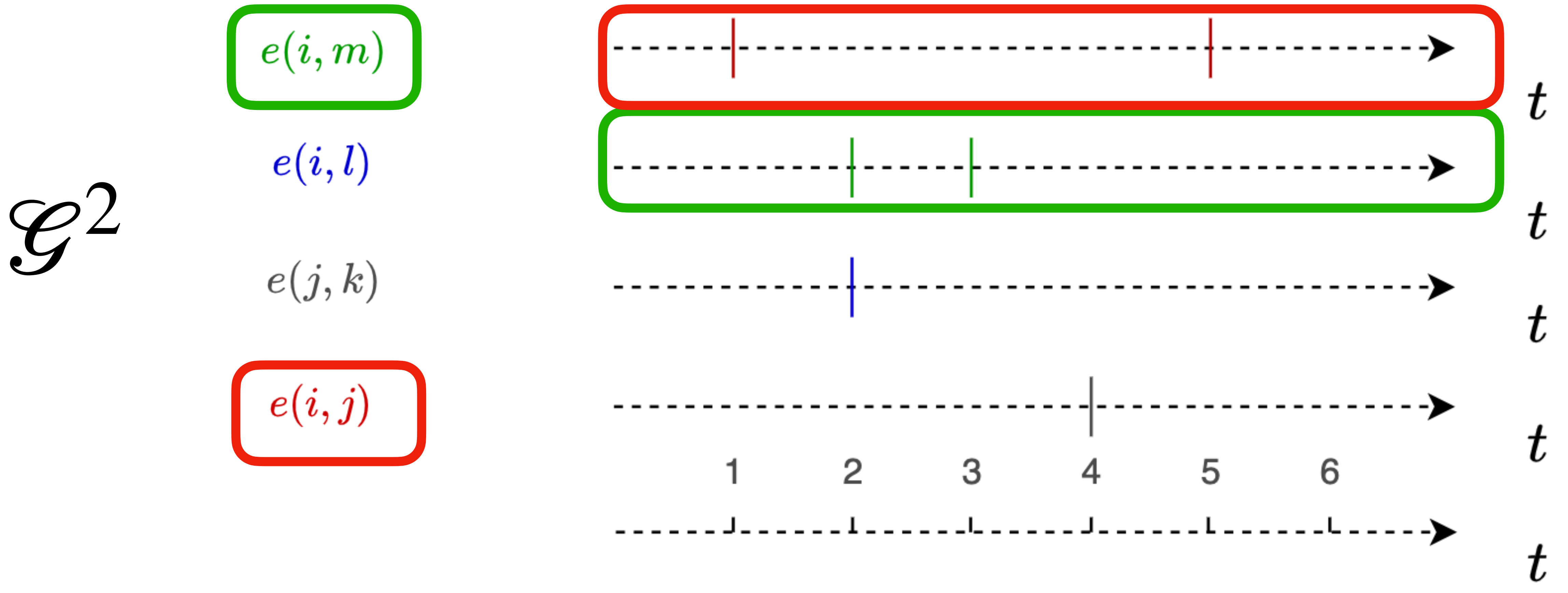
Randomised reference models



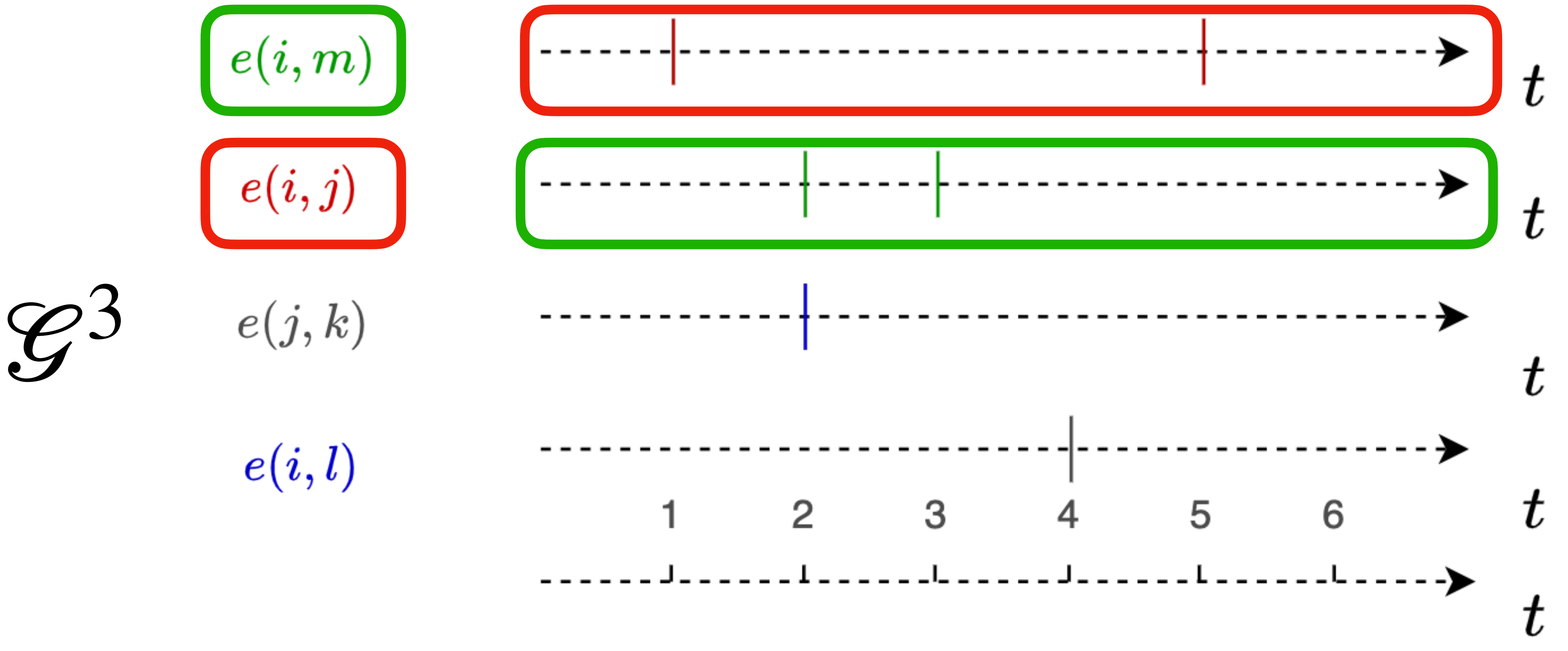
$\mathcal{G}^1$

Link	$\mathcal{G}$	$\mathcal{G}^1$
$e(i,j)$	1	5
$e(i,m)$	2	4
$e(i,l)$	2	3
$e(i,m)$	3	1
$e(j,k)$	4	2
$e(i,j)$	5	2

reshuffle timestamps



swap time series of links



swap time series of links with same number of activations

Gauvin, Laetitia, et al. "Randomized reference models for temporal networks." *arXiv preprint arXiv:1806.04032* (2018).

Topological and temporal distance of contacts

- **Topological** distance of two contacts:

$$\eta(\ell(i,j,t), \ell(k,l,s)) = \begin{cases} \min_{u \in \{i,j\},\ v \in \{k,l\}} (h(u,v) + 1) & e(i,j) \neq e(k,l) \\ 0 & e(i,j) = e(k,l) \end{cases}$$

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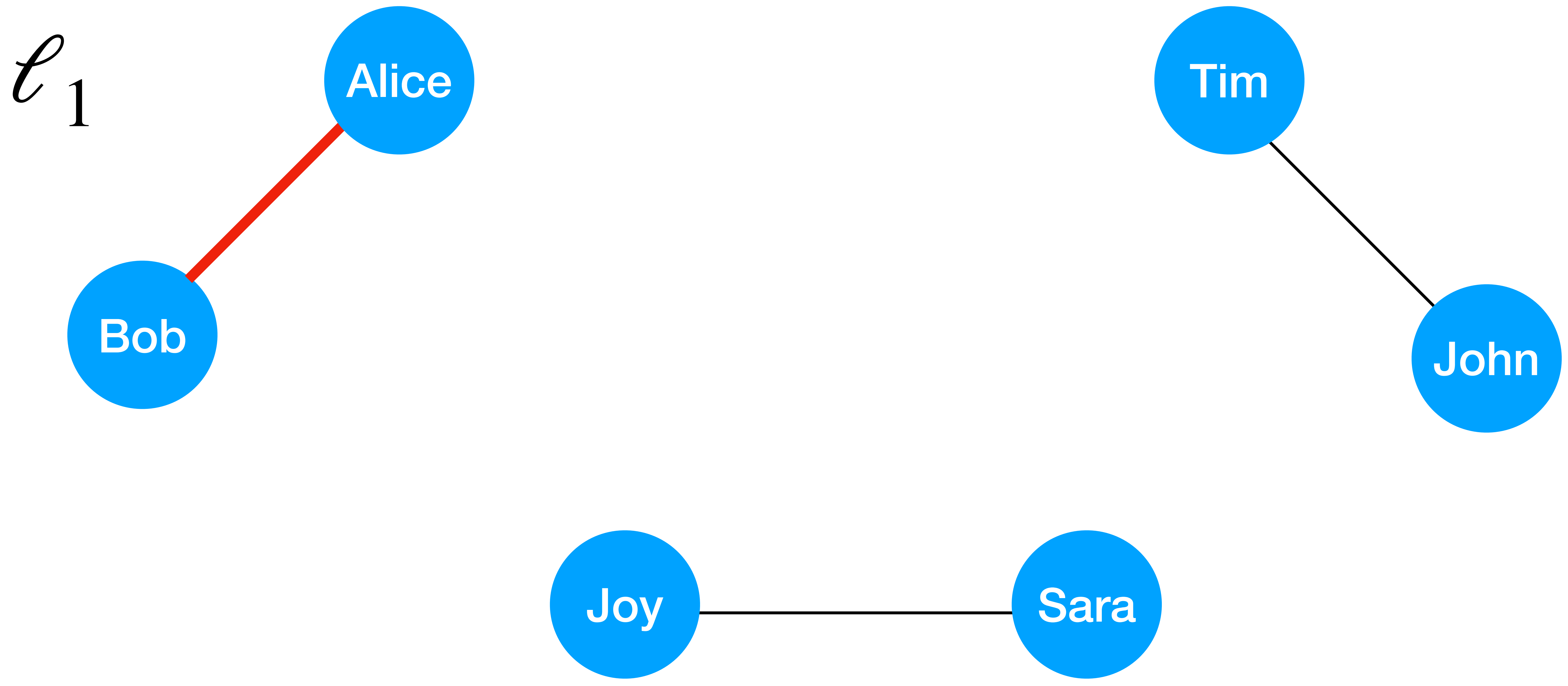
- **Temporal** distance of two contacts:

$$\mathcal{T}(\ell(i,j,t),\ell(k,l,s)) = |t-s|$$

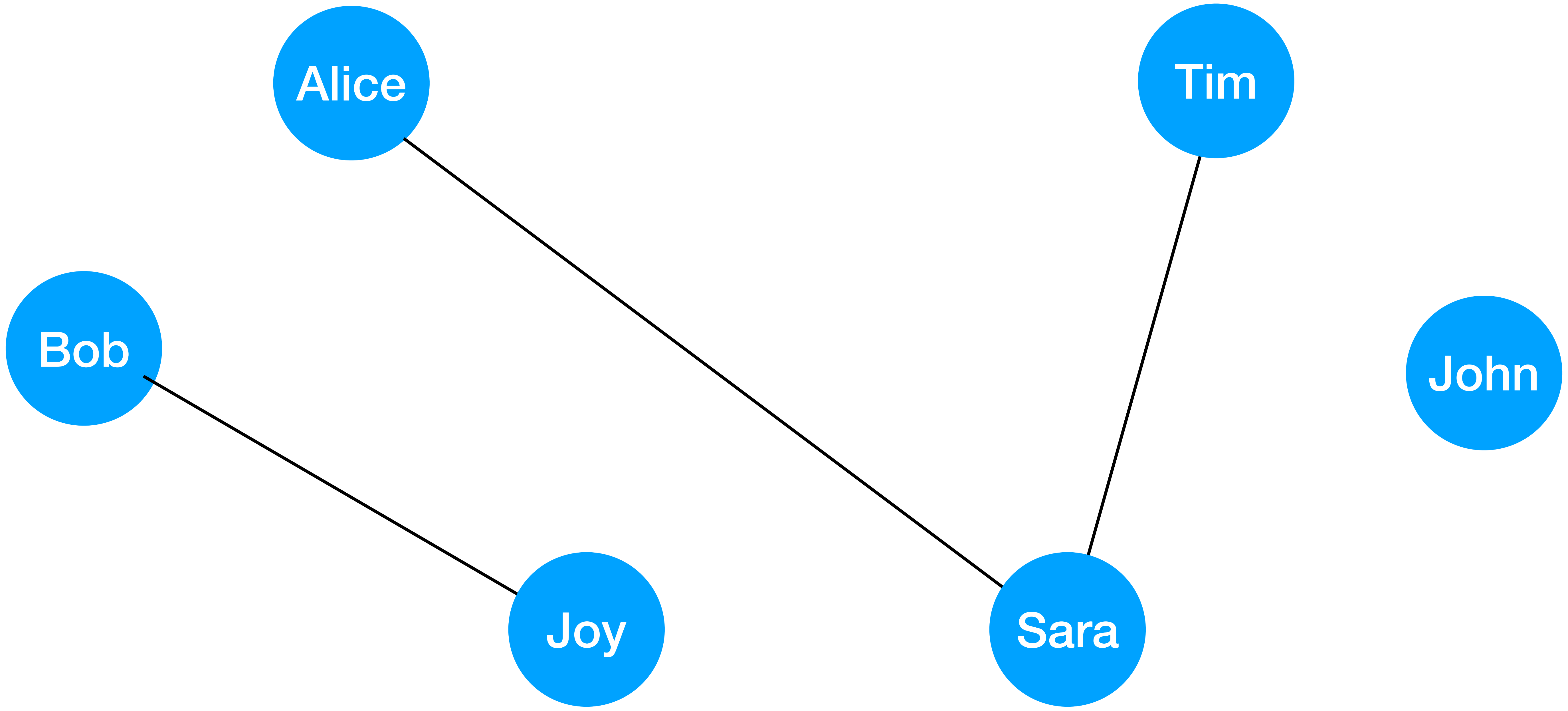
Example

h 18:00



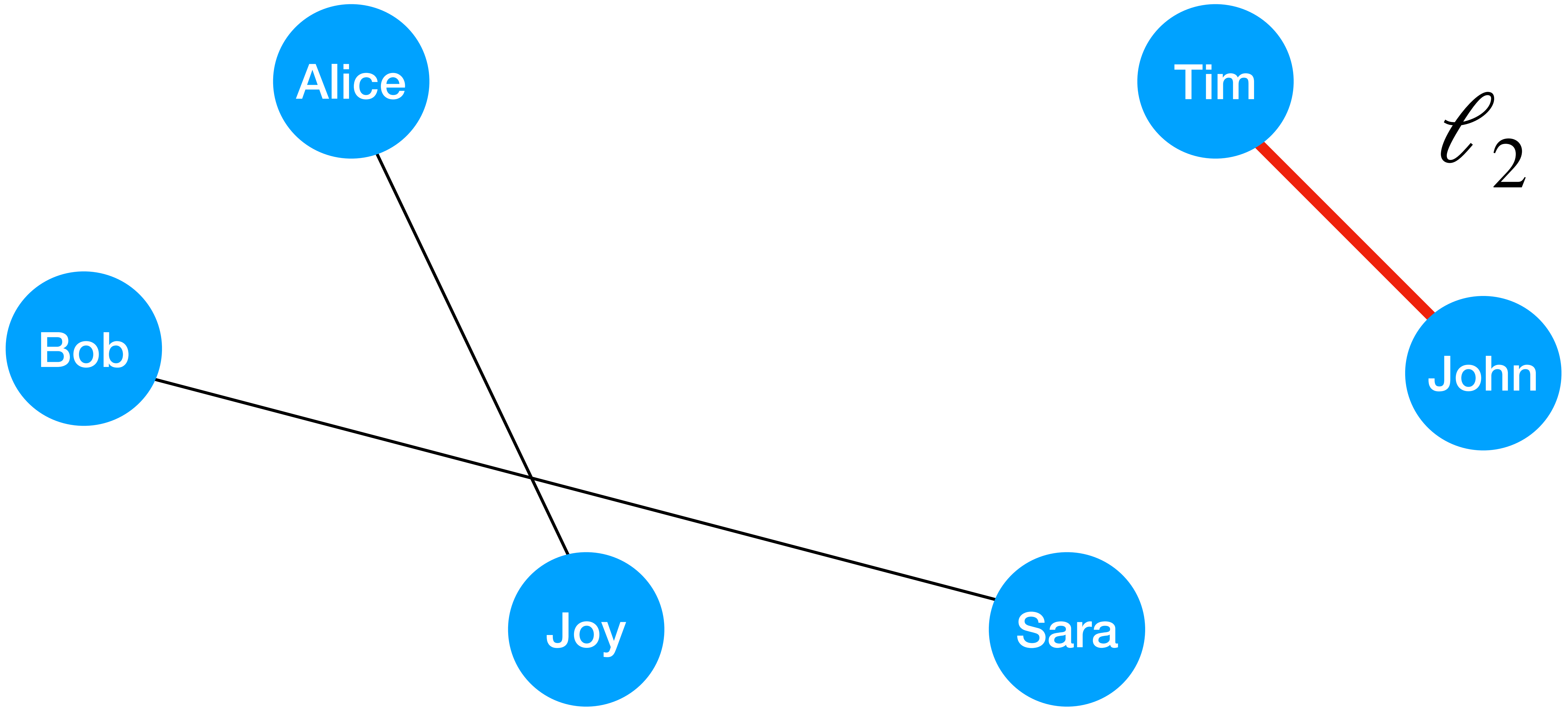
Example

h 18:30

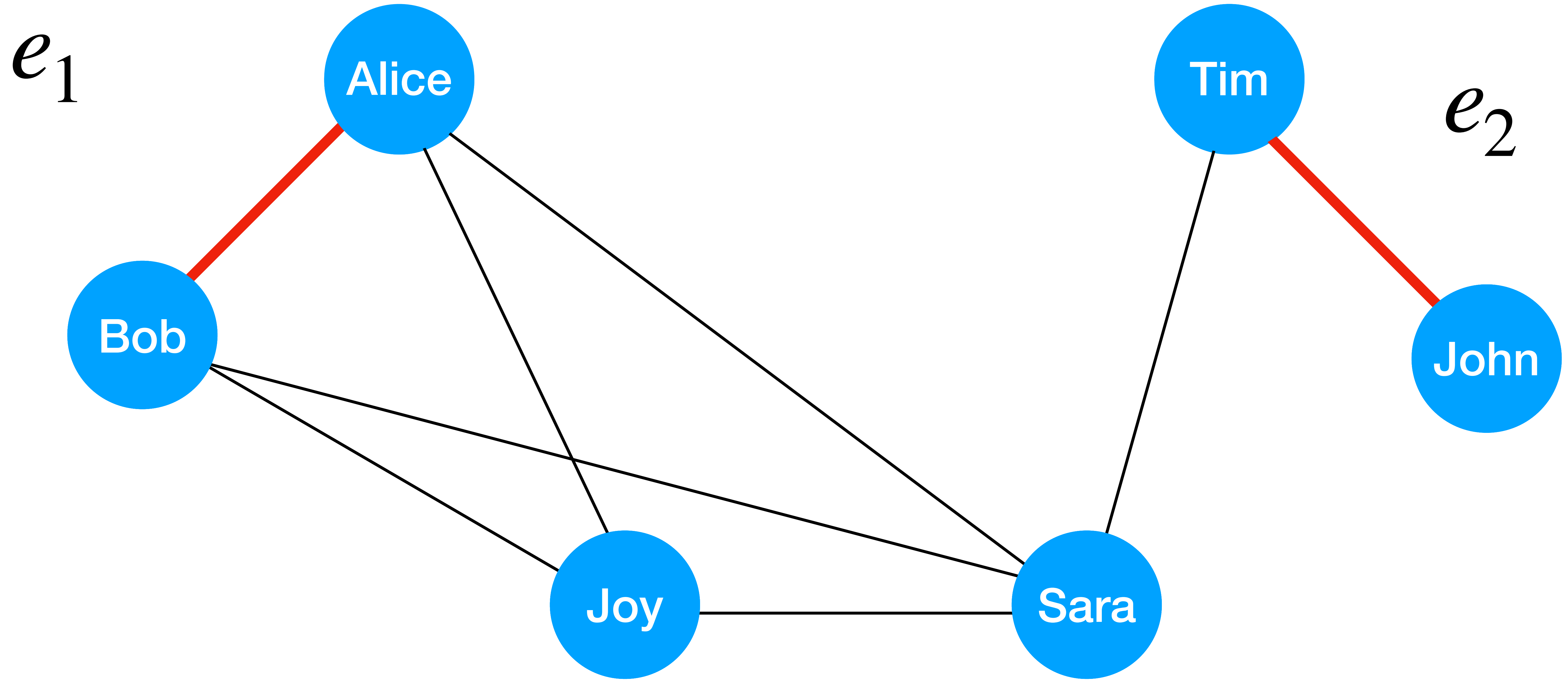


Example

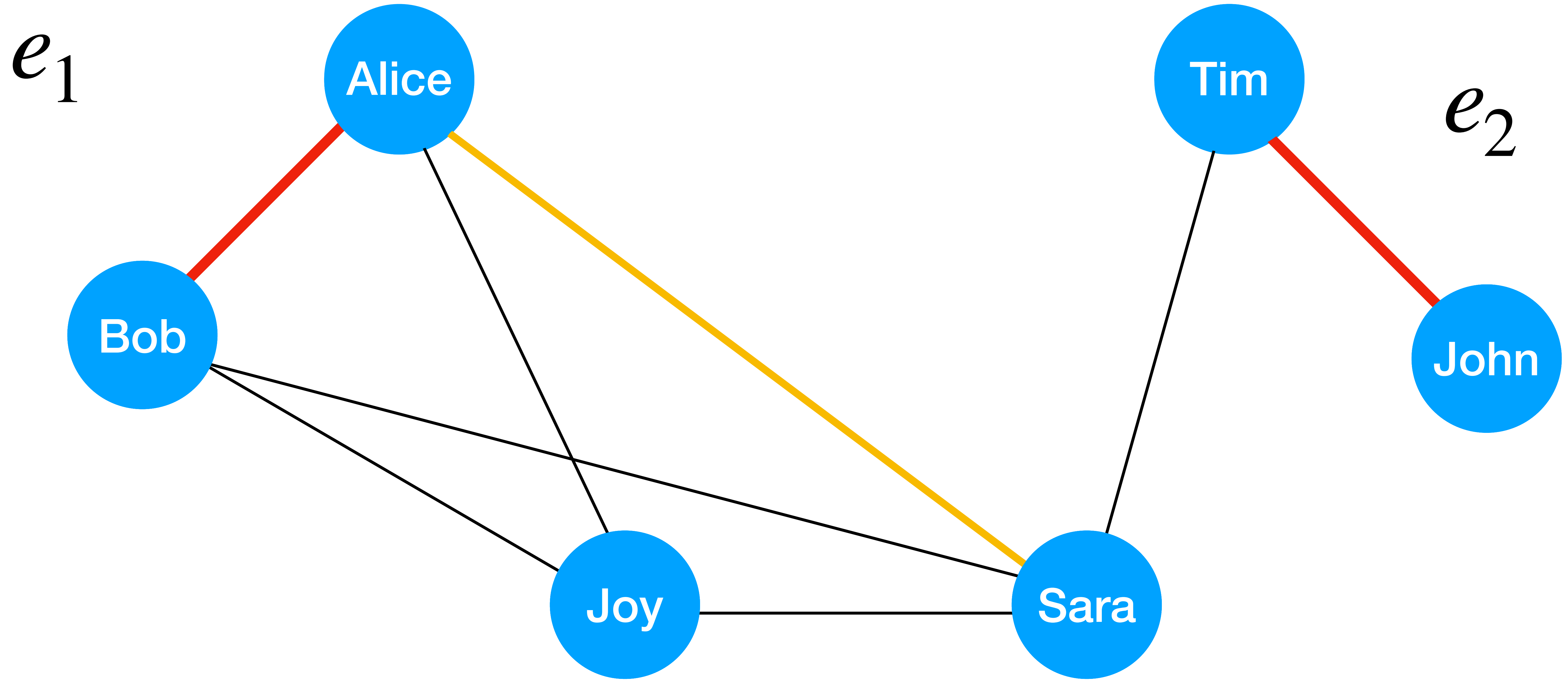
h 19:00



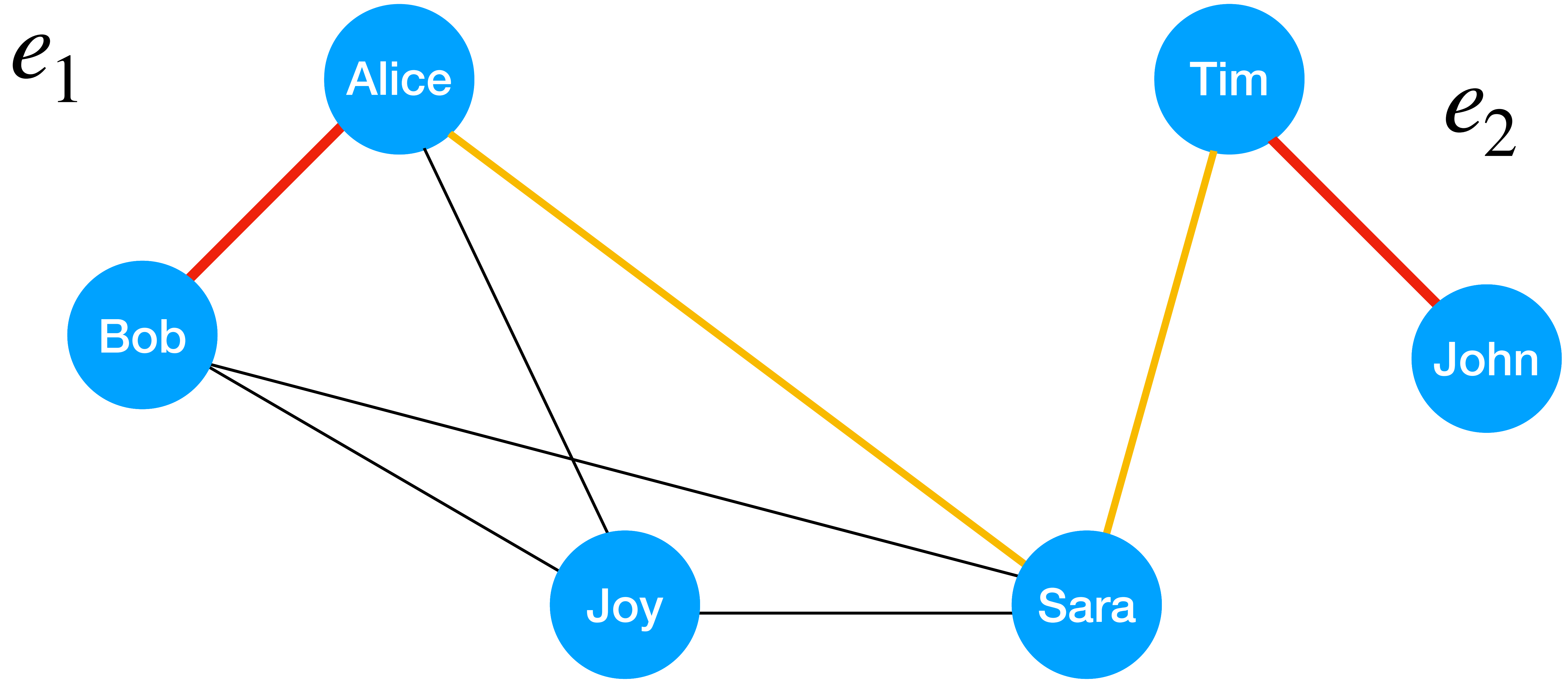
Example



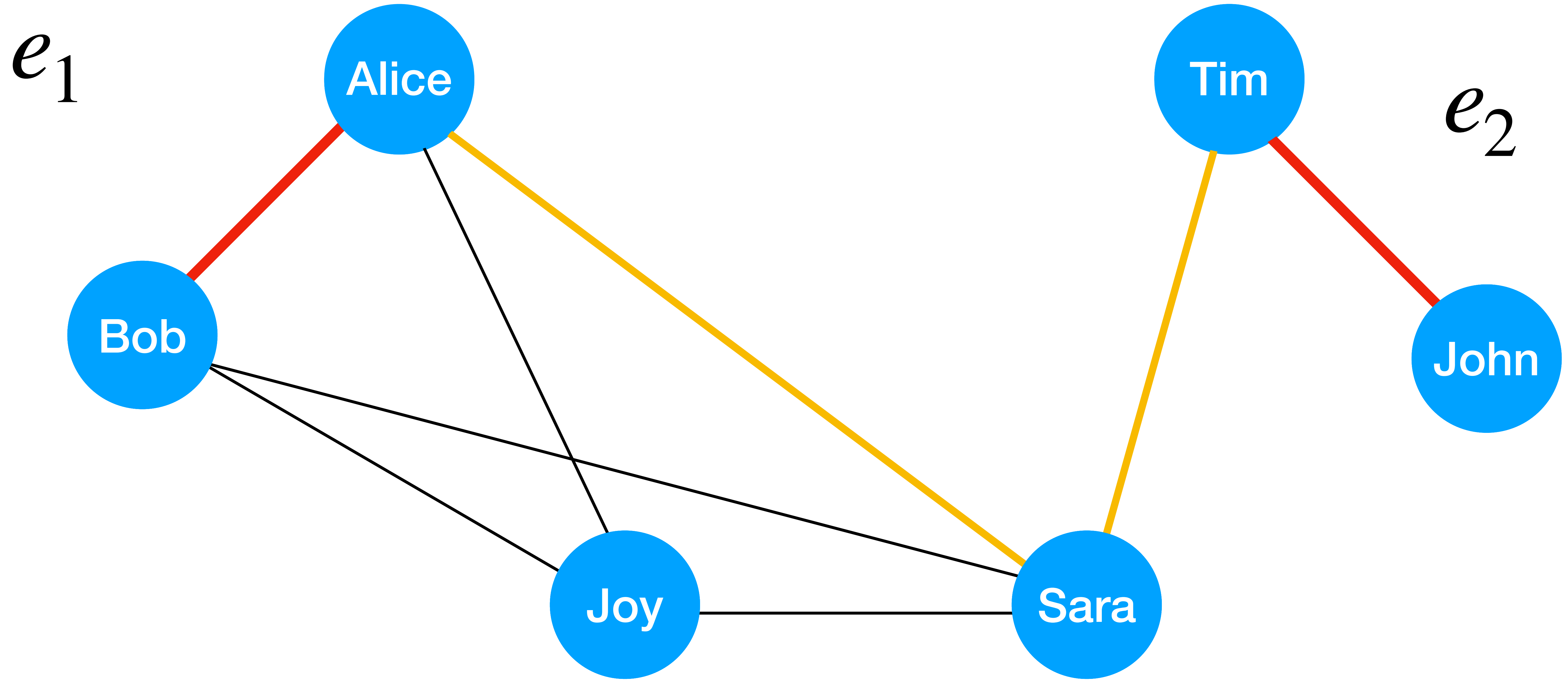
Example



Example

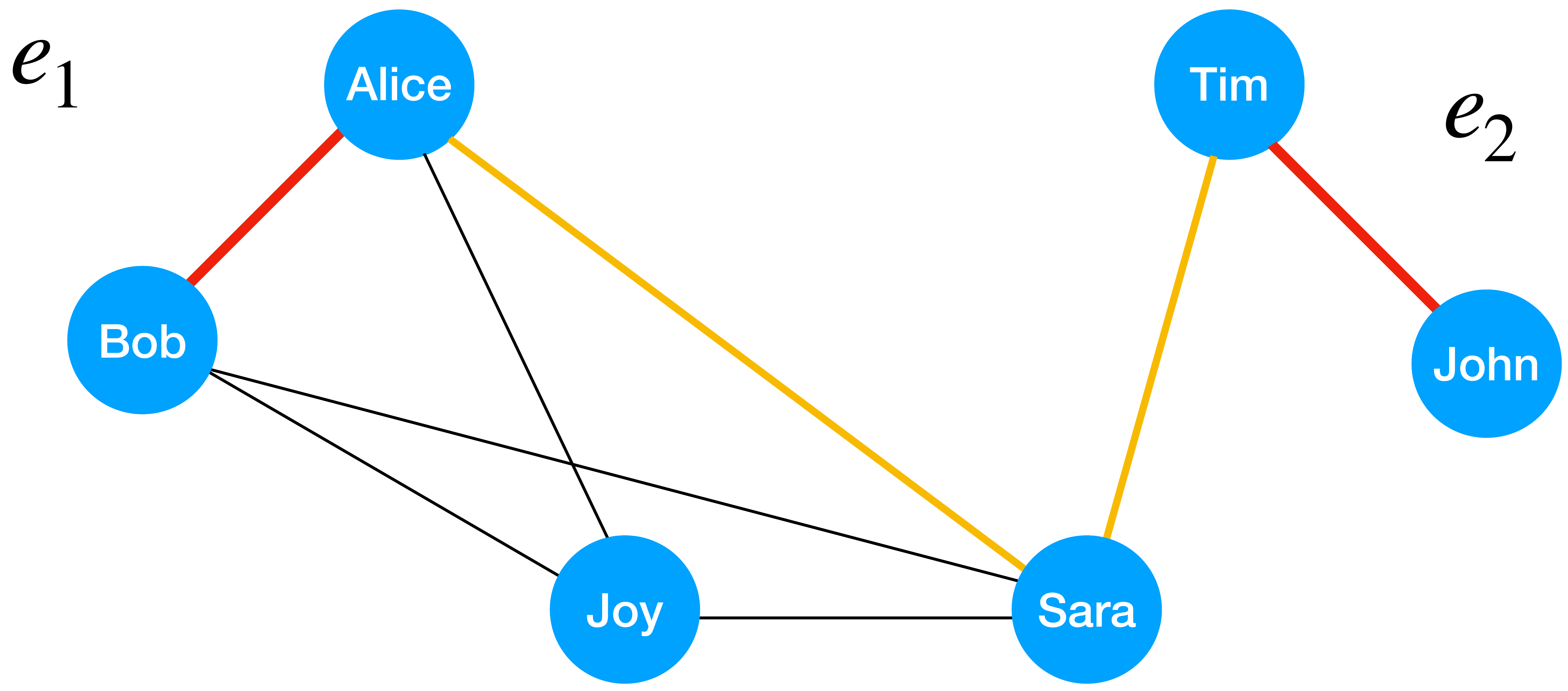


Example



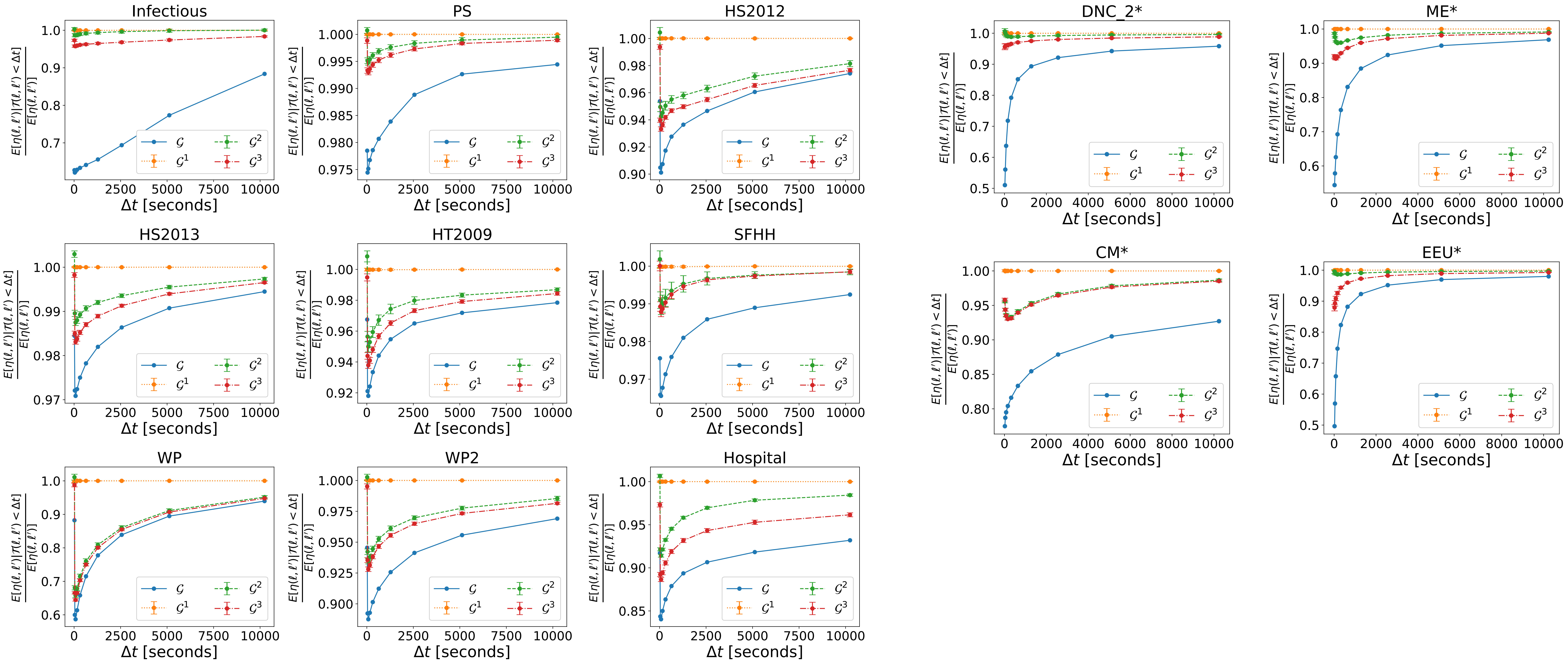
Topological distance: $\eta(\ell_1, \ell_2) = 3$

Example

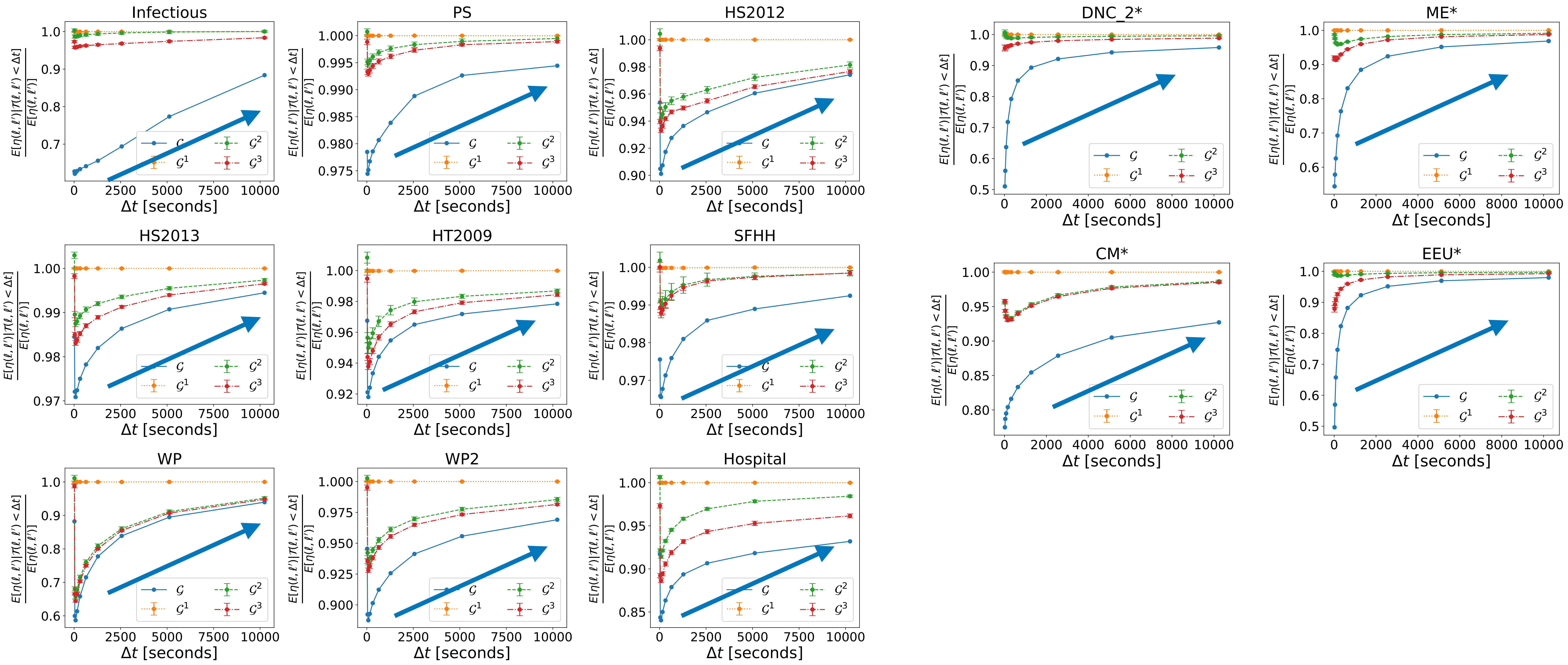


Topological distance: $\eta(\ell_1, \ell_2) = 3$ **Temporal distance:** $\mathcal{T}(\ell_1, \ell_2) = 1h$

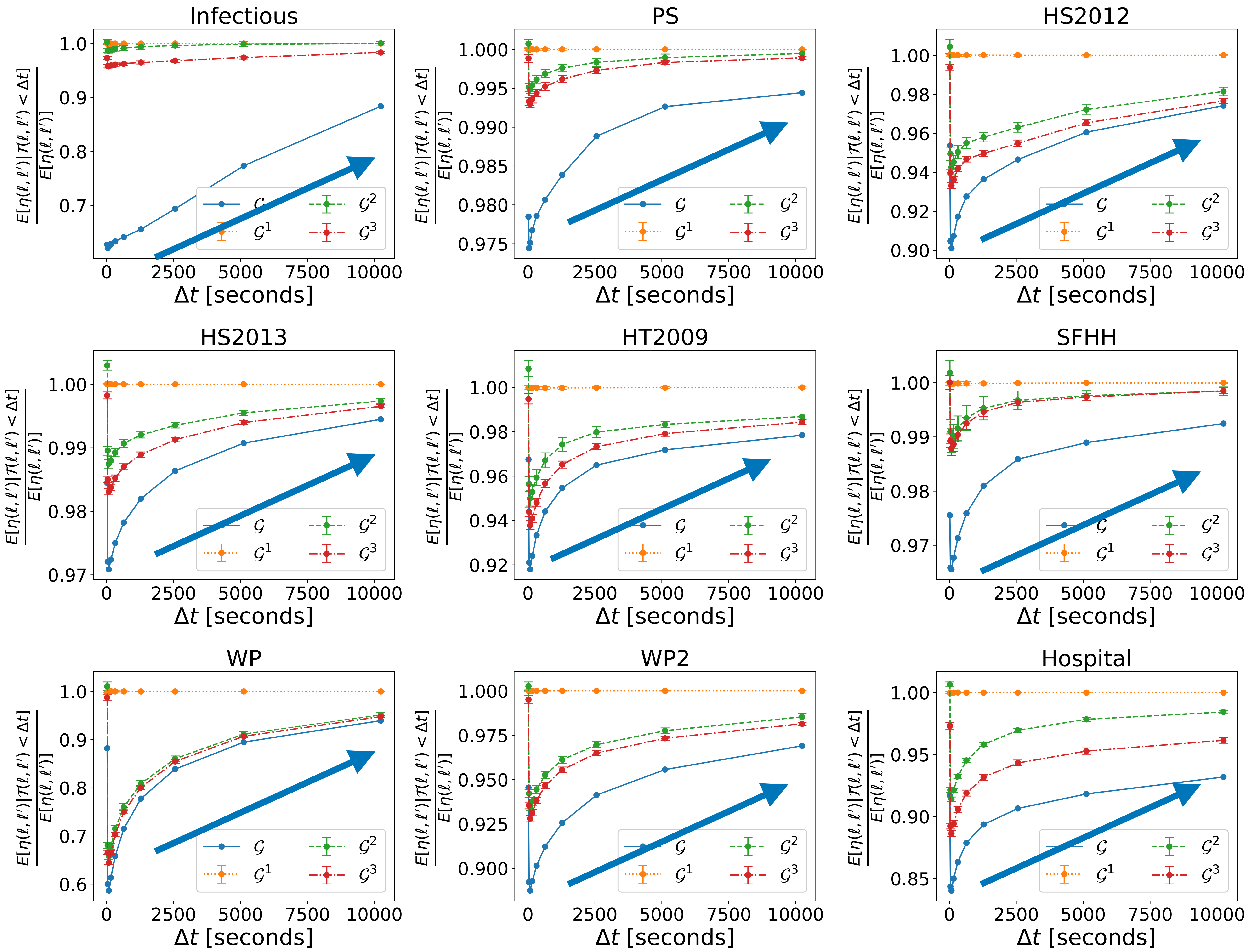
Interrelation of topological and temporal distance of contacts

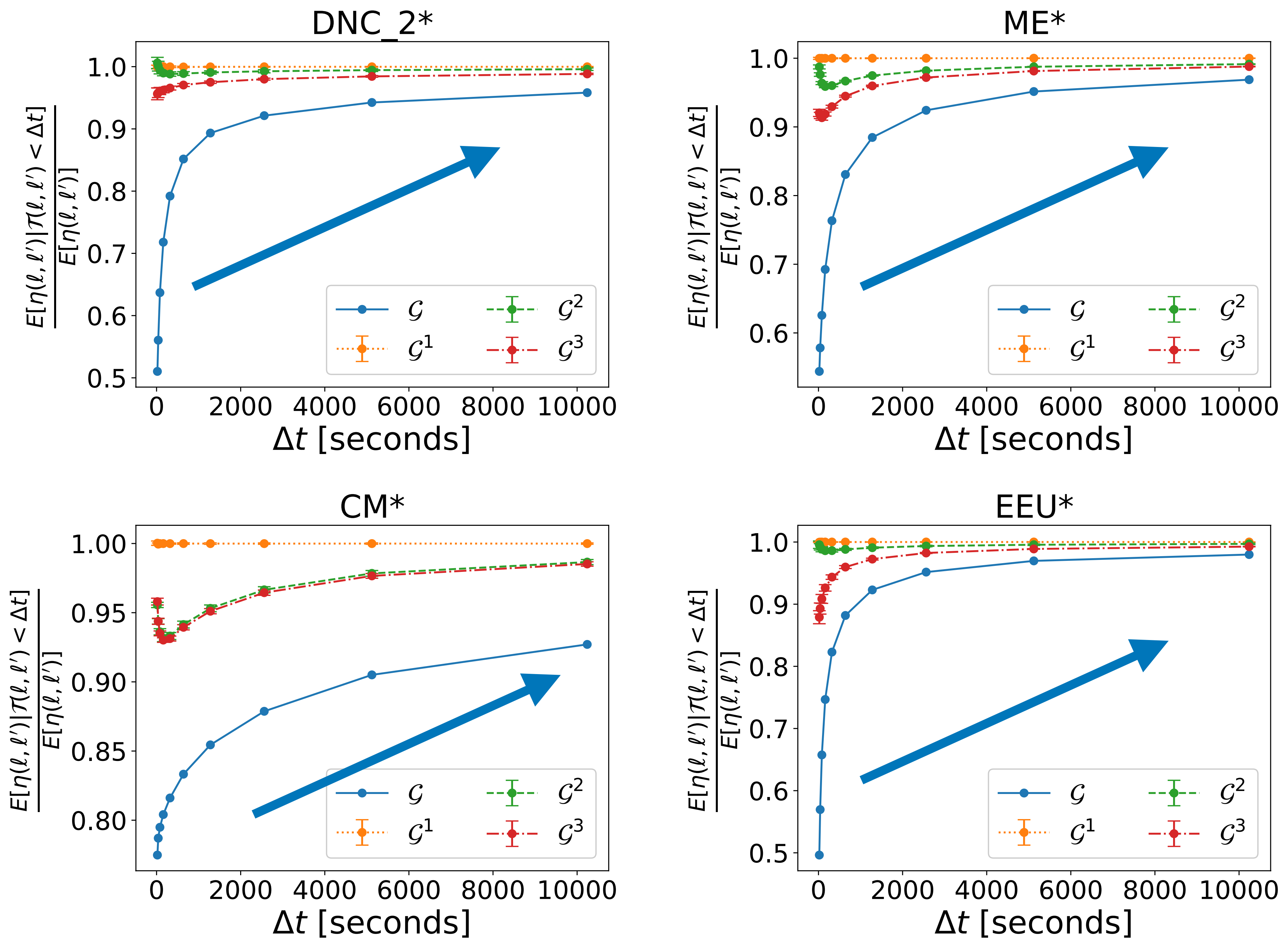


Interrelation of topological and temporal distance of contacts



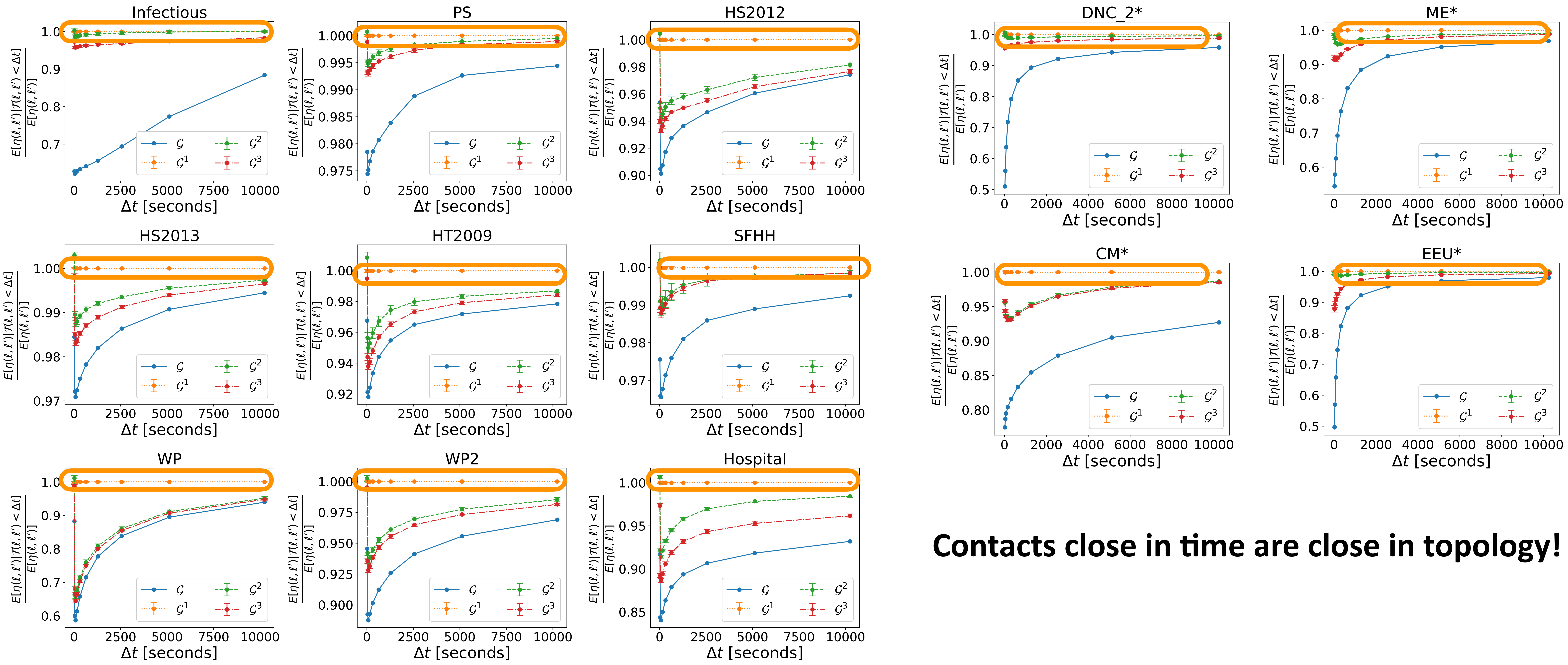
Interrelation of topological and temporal distance of contacts





Contacts close in time are close in topology!

Interrelation of topological and temporal distance of contacts



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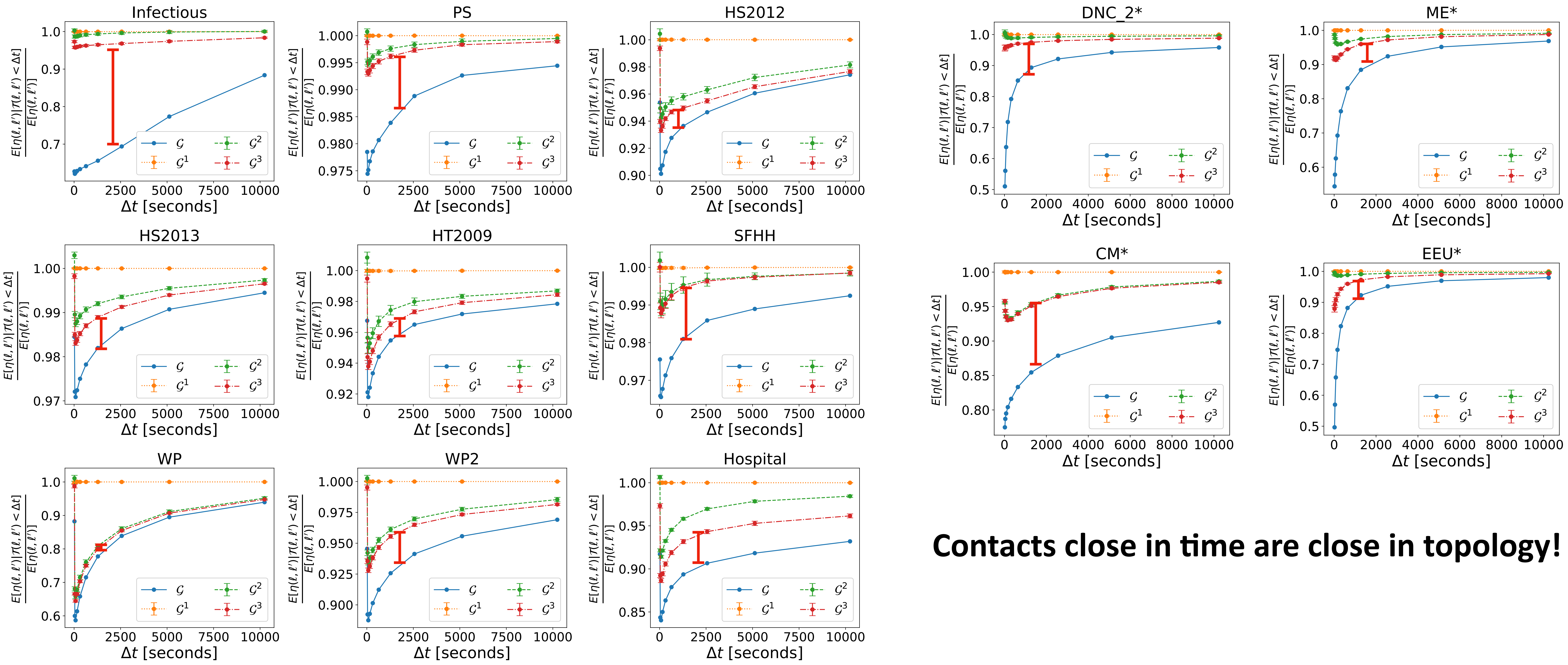
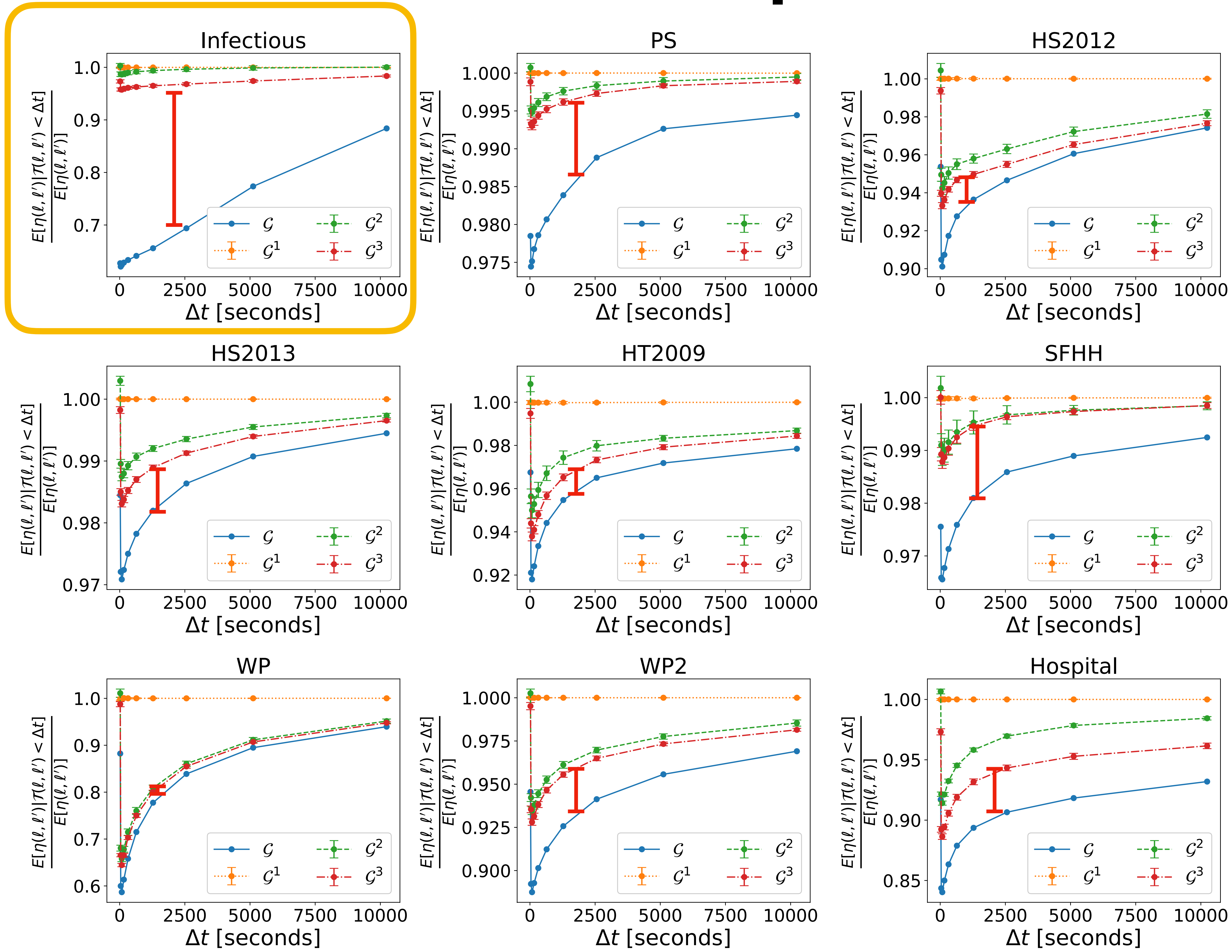
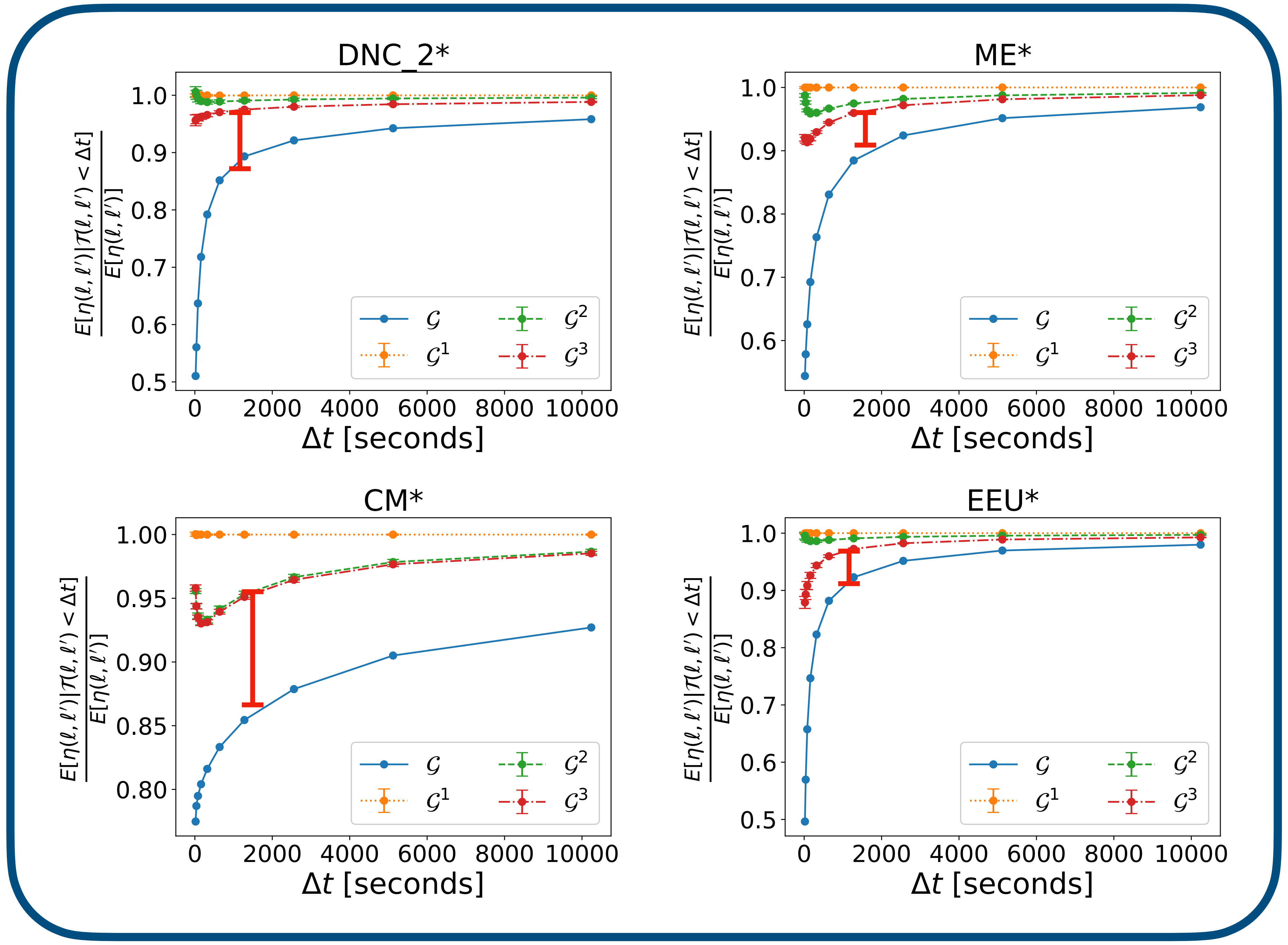
Interrelation of topological and temporal distance of contacts



Contacts close in time are close in topology!

Interrelation of topological and temporal distance of contacts





Contacts close in time are close in topology!

Temporal properties of single link activity cannot explain these trends, especially in virtual contacts and Infectious!

Local analysis of temporal correlation around a link

Local analysis around a link
=
Link egonetwork

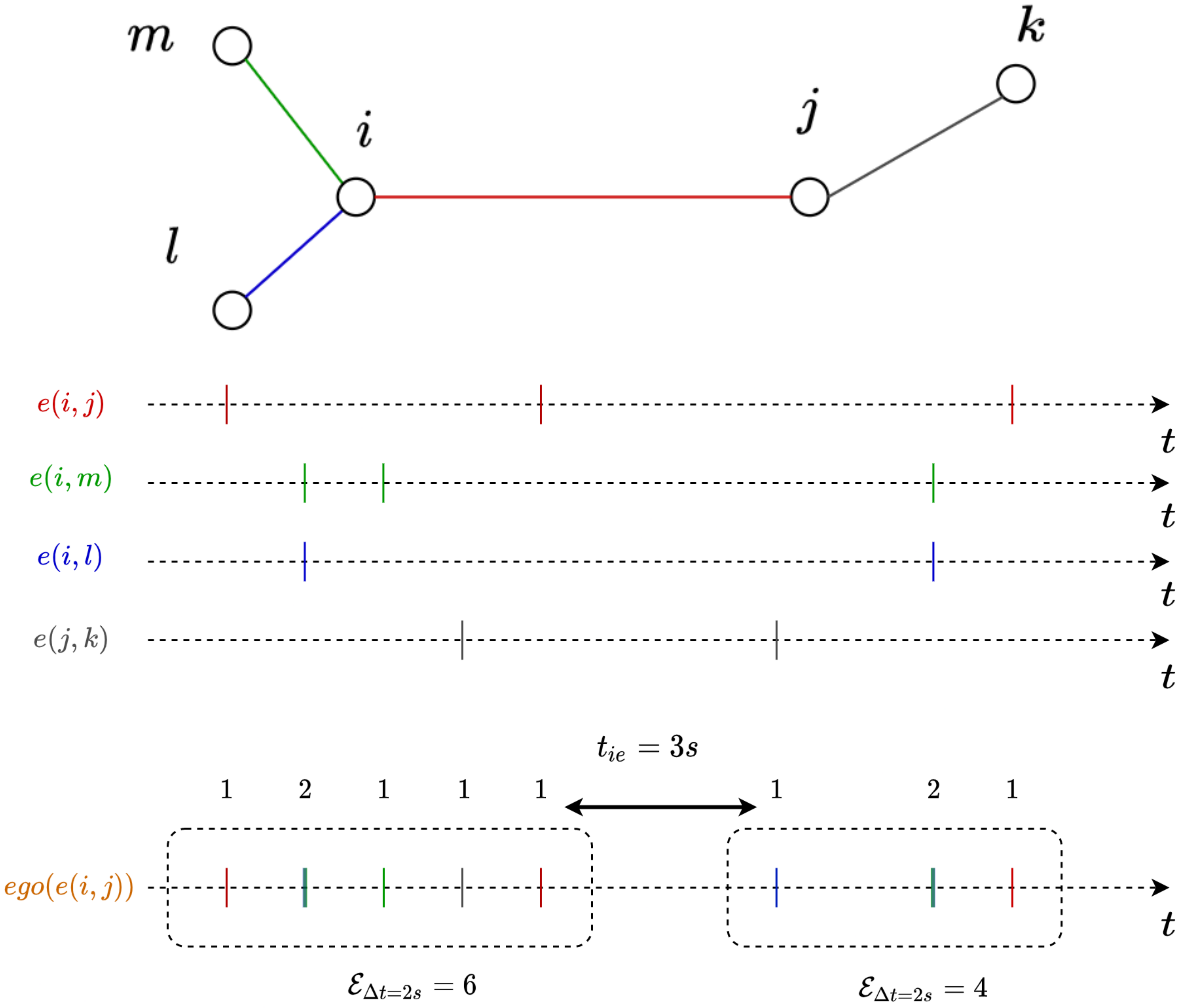
Marton Karsai, Kimmo Kaski, Albert-László Barabási, and János Kertész.
Universal features of correlated bursty behaviour. Scientific Reports, 2:397, 2012.

Local analysis of temporal correlation around a link

Local analysis around a link

=

Link egonetwork



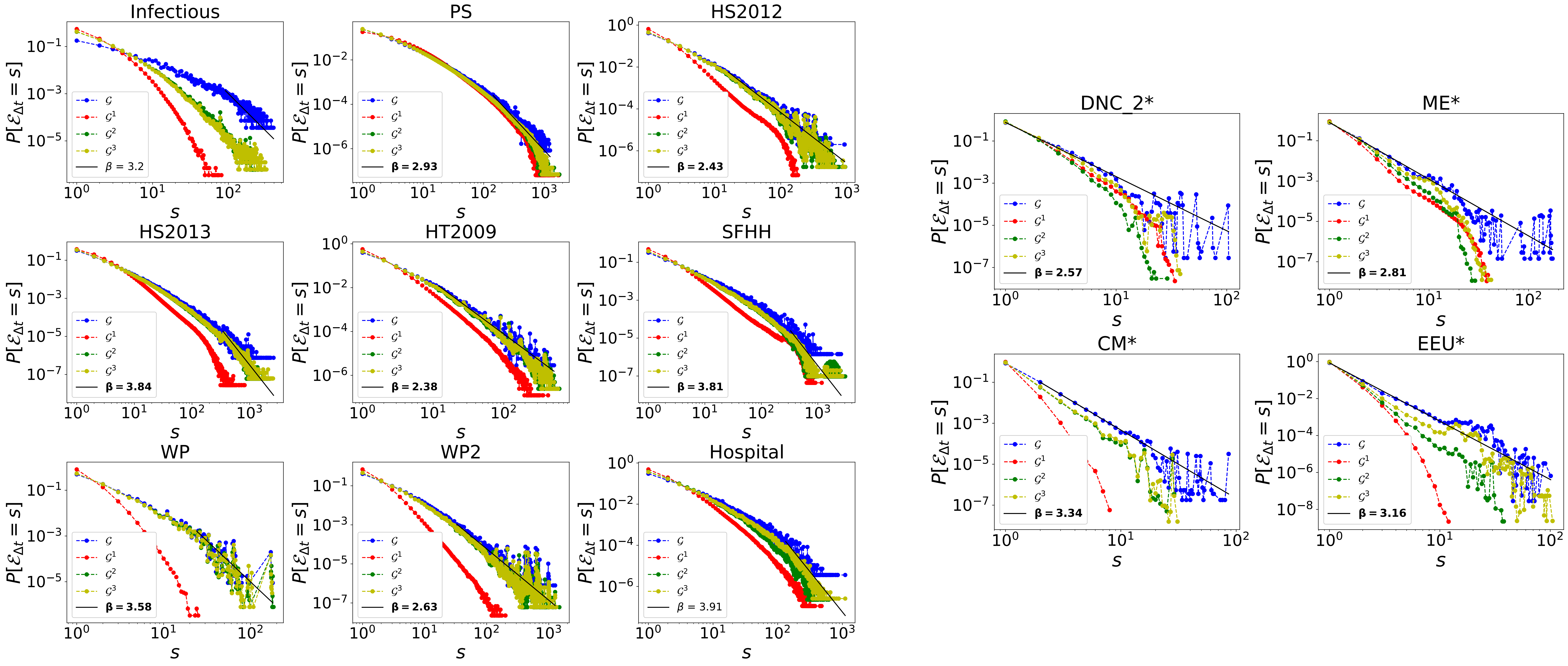
Temporal correlation of egonetwork activity

=

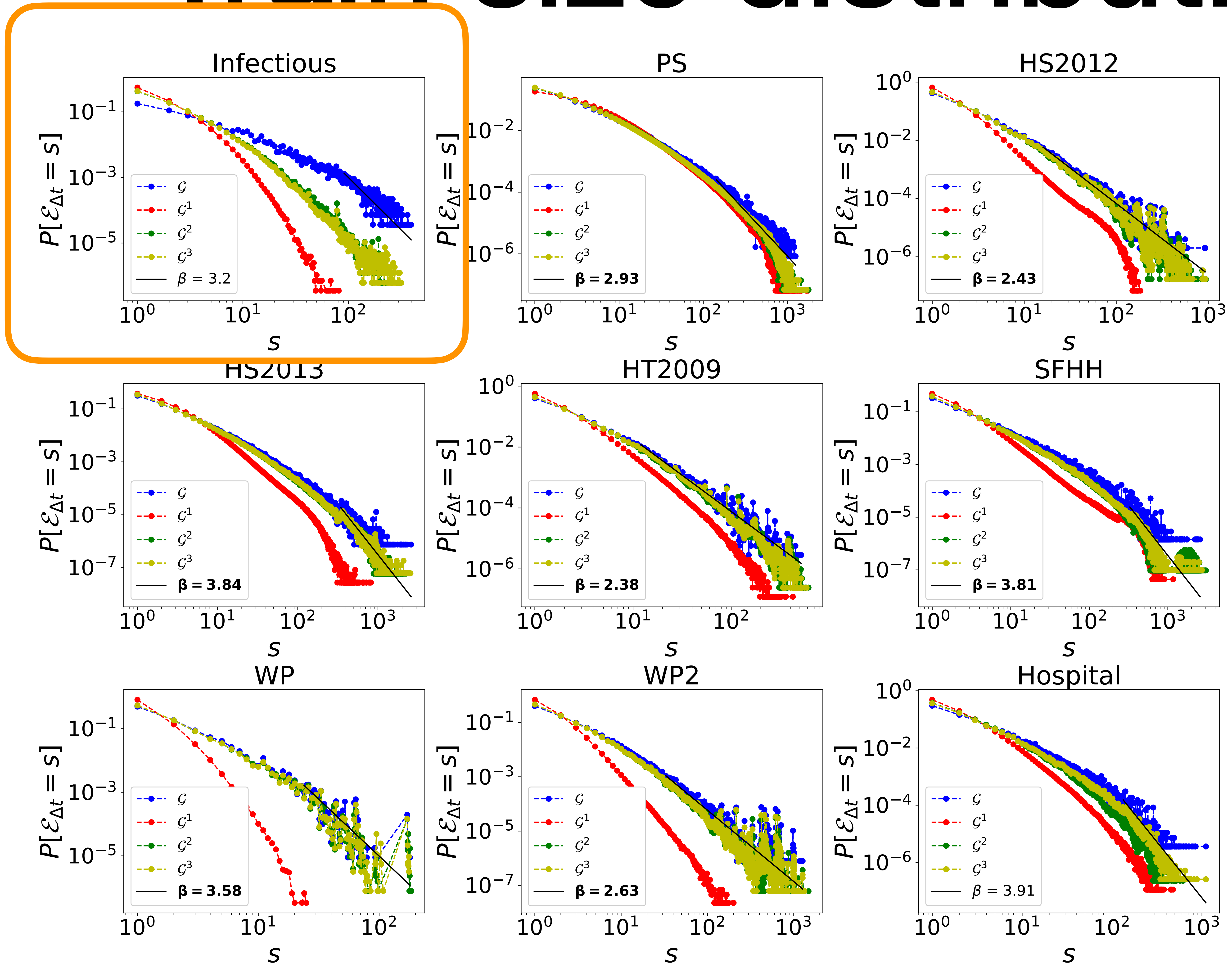
Train size distribution of egonetwork activity

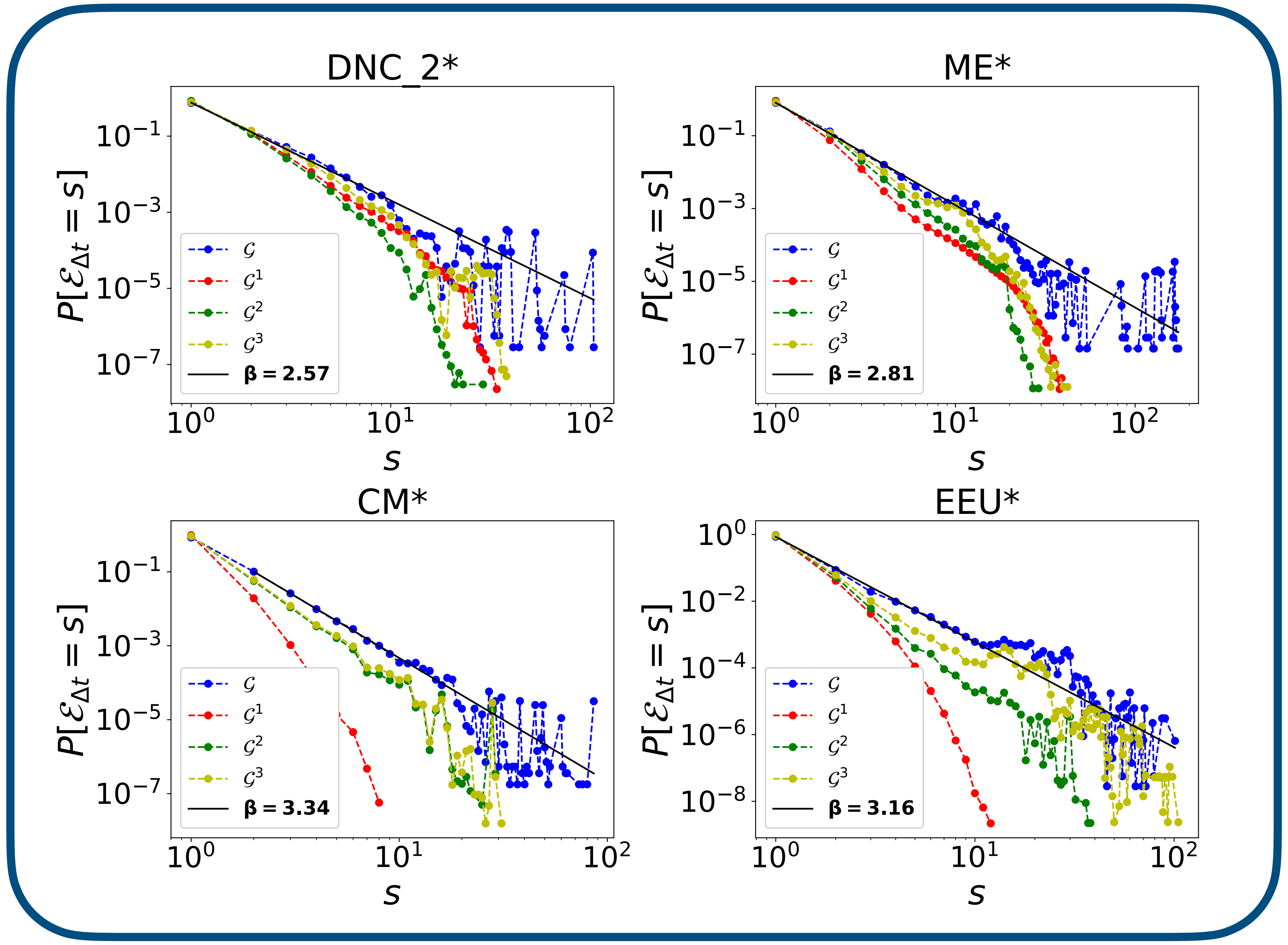
Marton Karsai, Kimmo Kaski, Albert-László Barabási, and János Kertész. Universal features of correlated bursty behaviour. Scientific Reports, 2:397, 2012.

Train size distribution with $\Delta t = 60s$

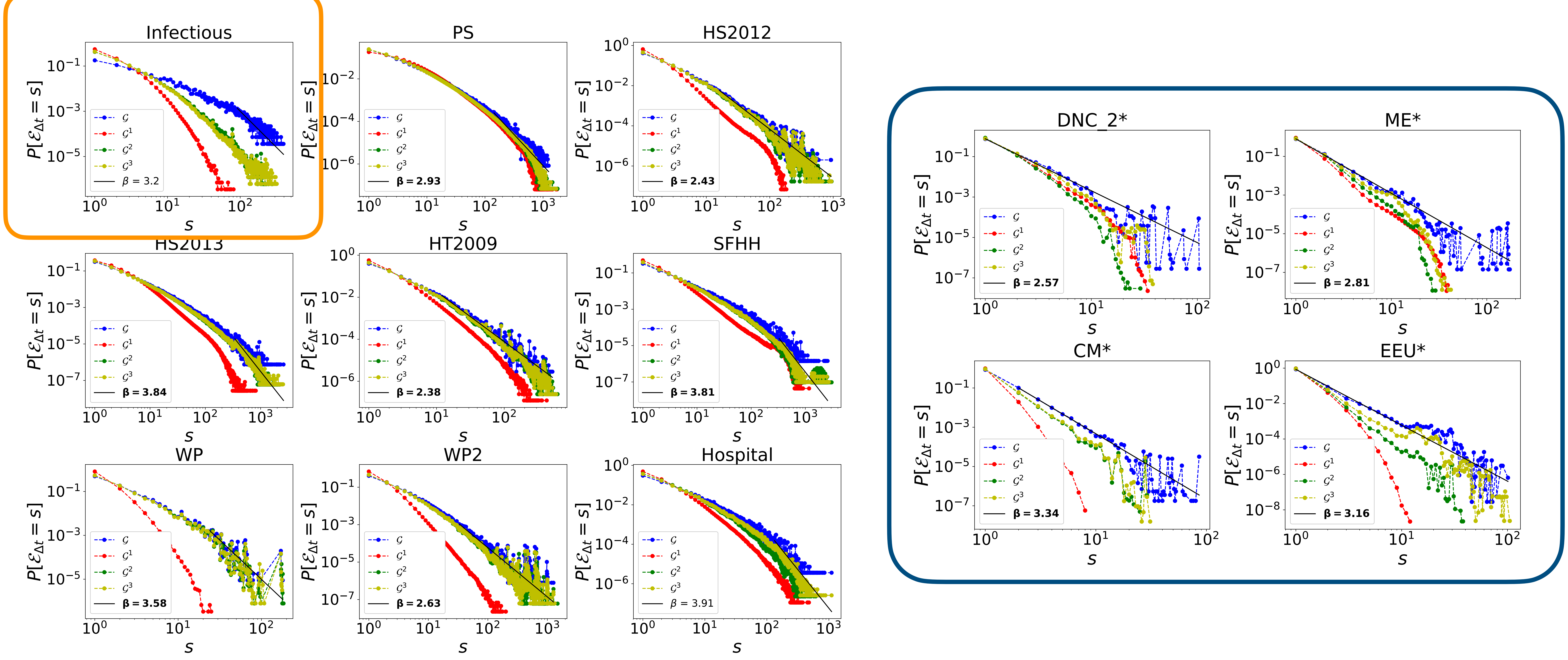


Train size distribution with $\Delta t = 60s$



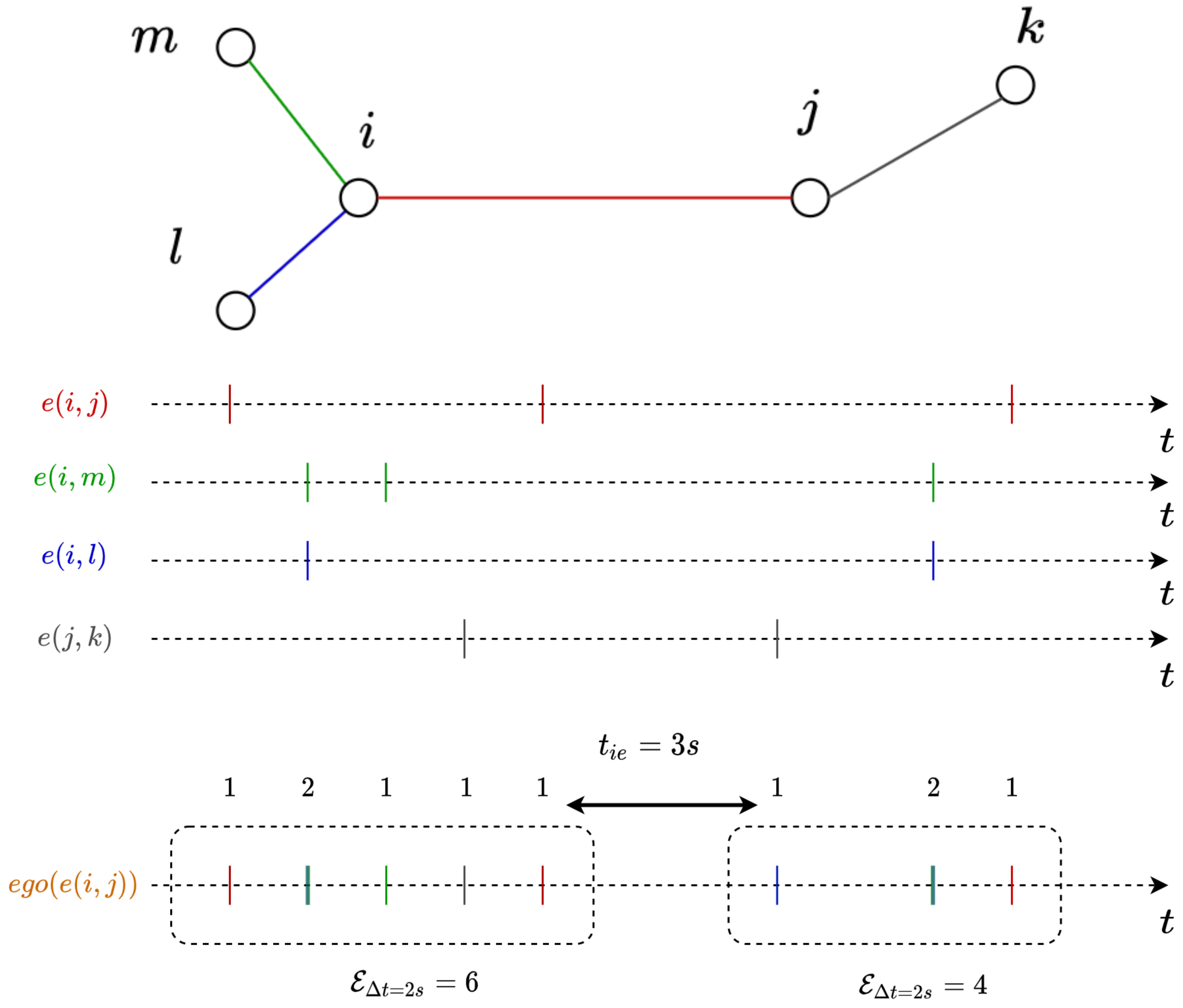


Train size distribution with $\Delta t = 60s$

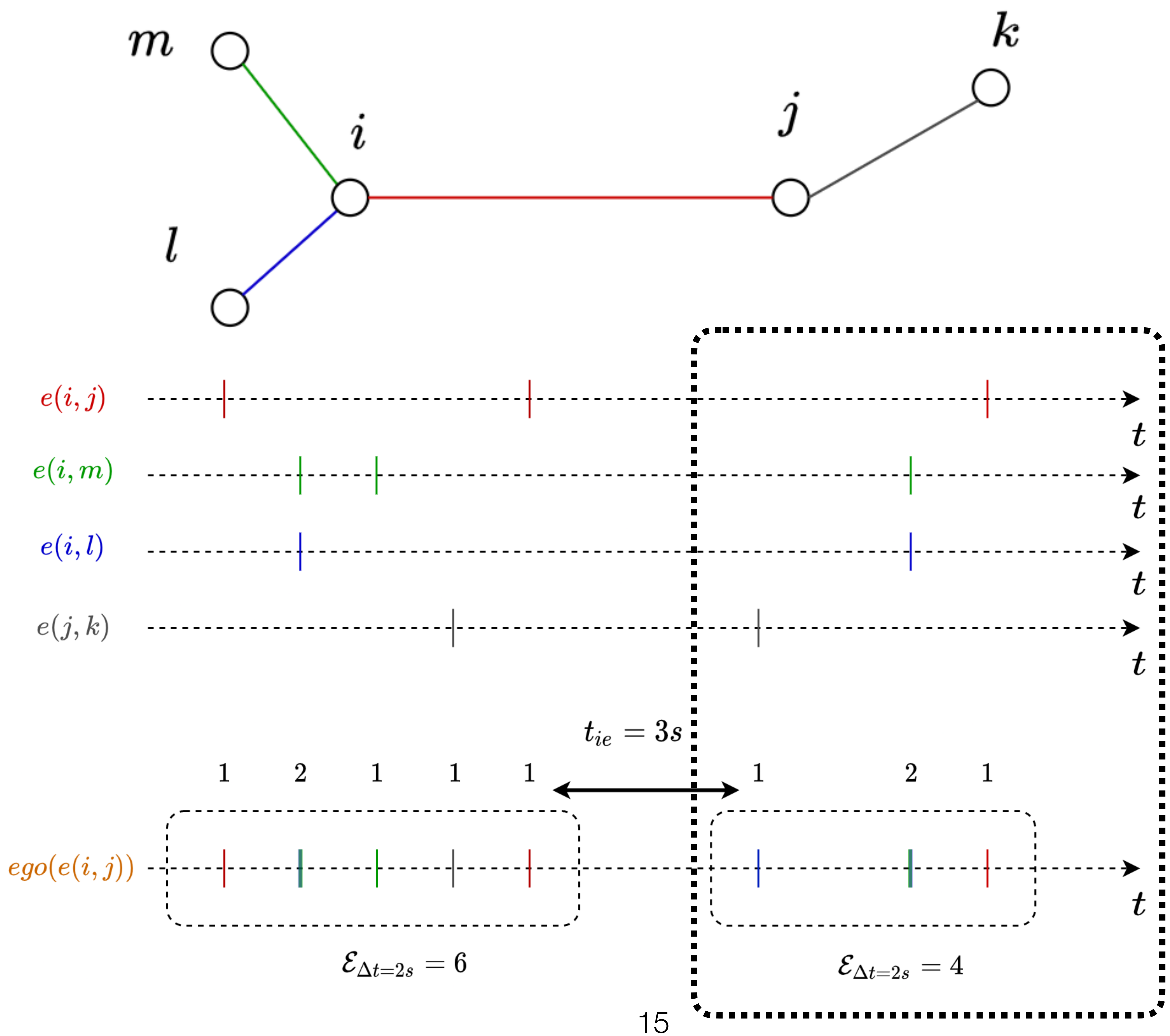


temporal correlation among neighbouring links = higher chance of long trains

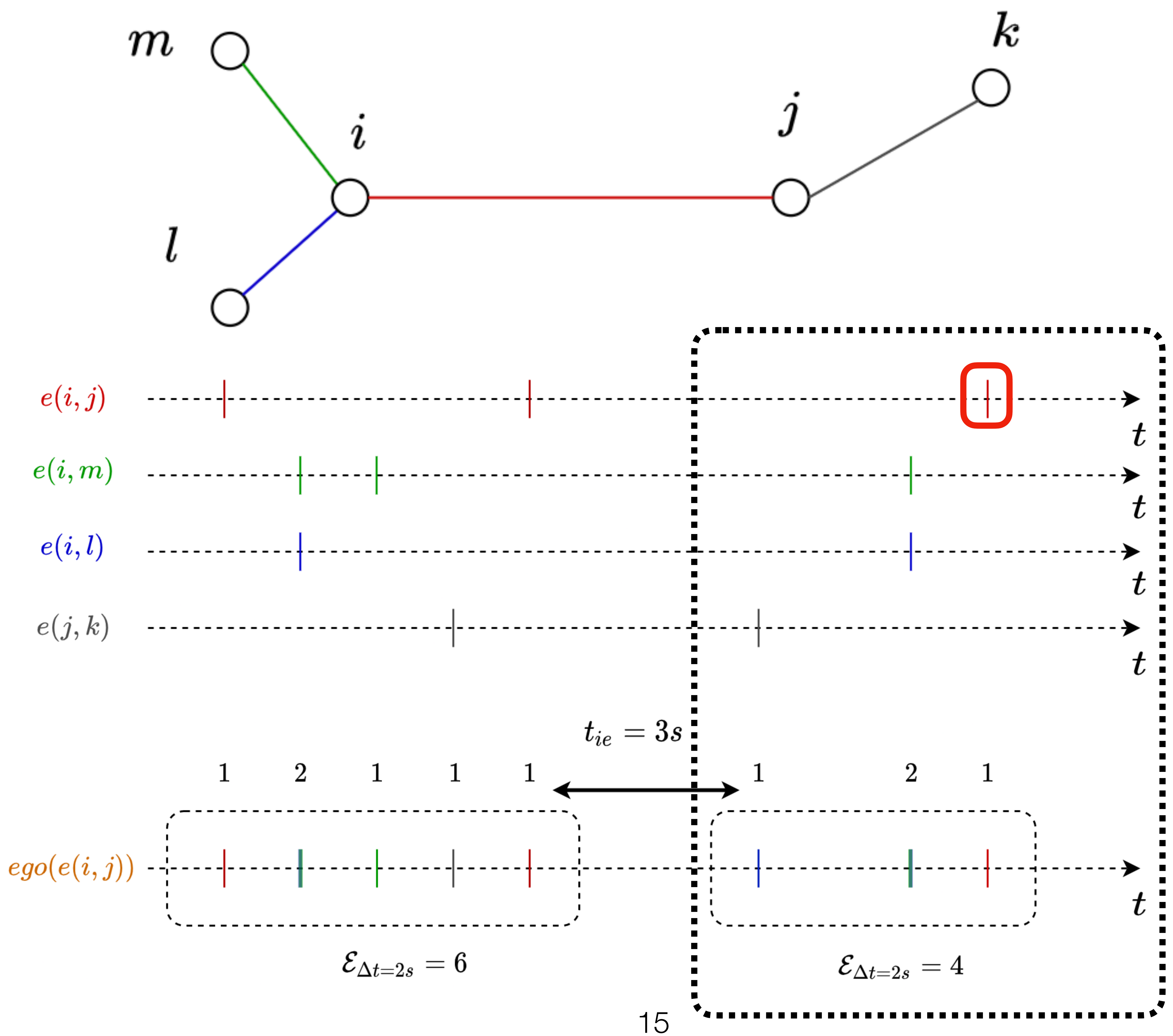
Number of active links during a train



Number of active links during a train



Number of active links during a train



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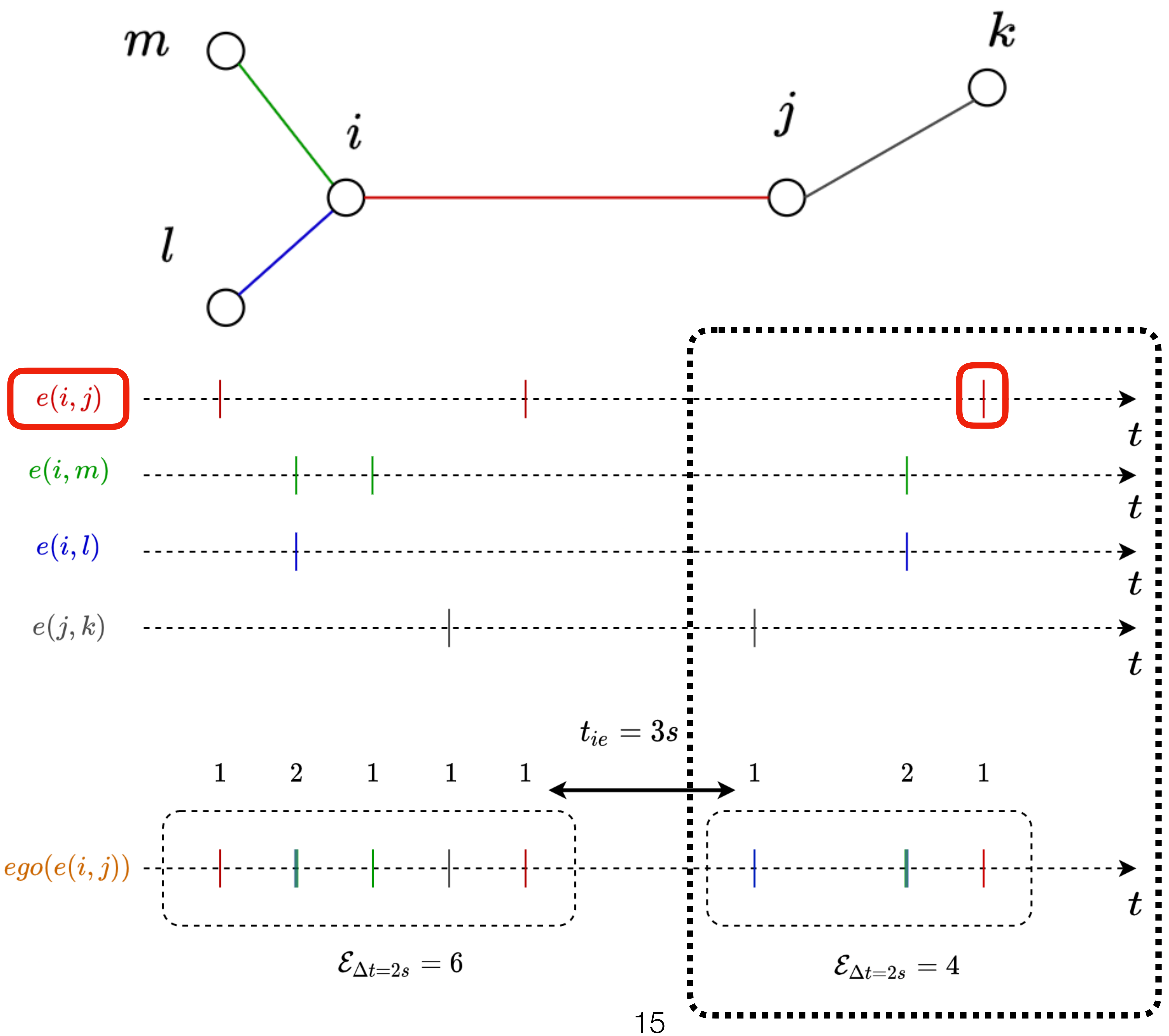

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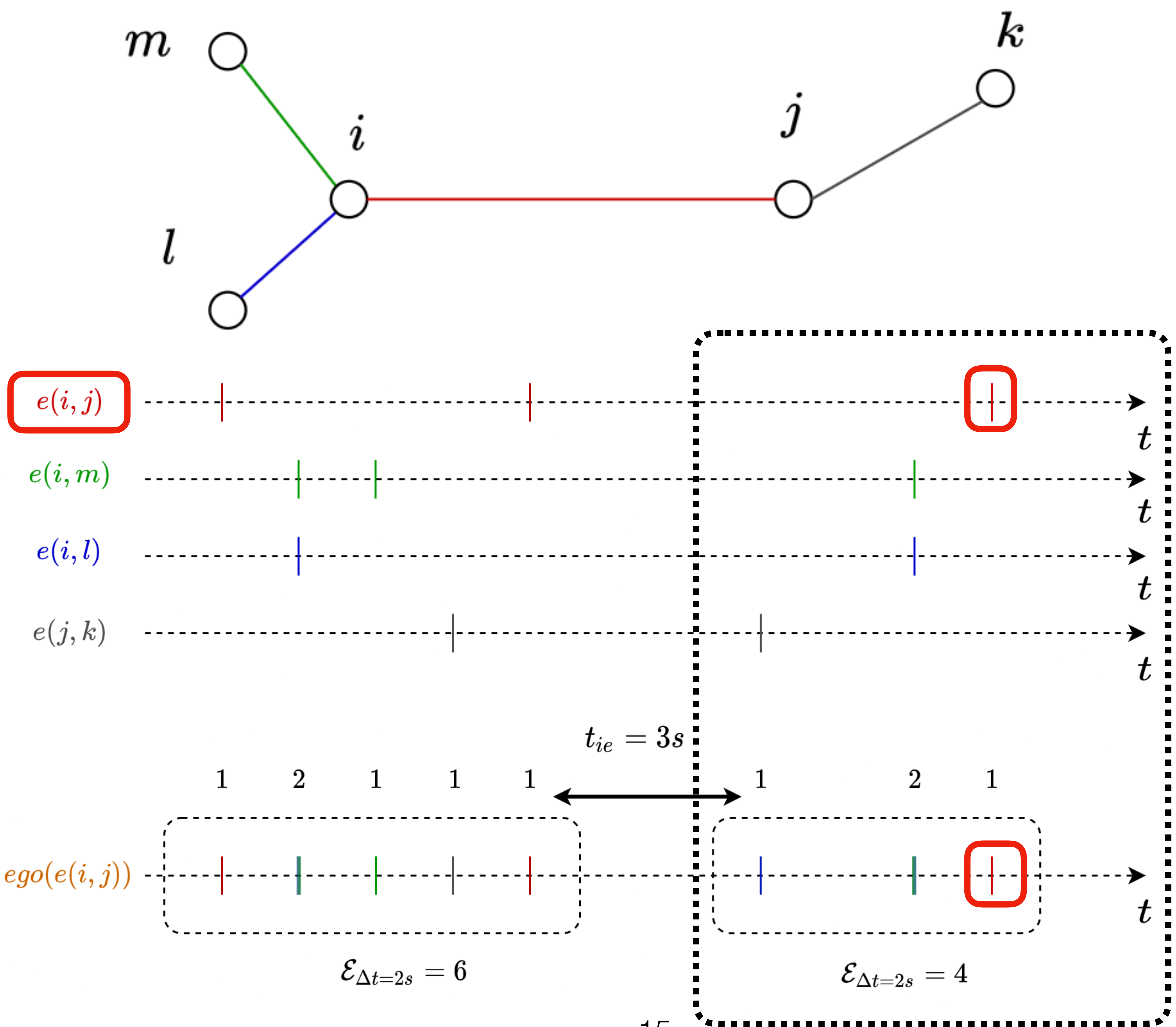

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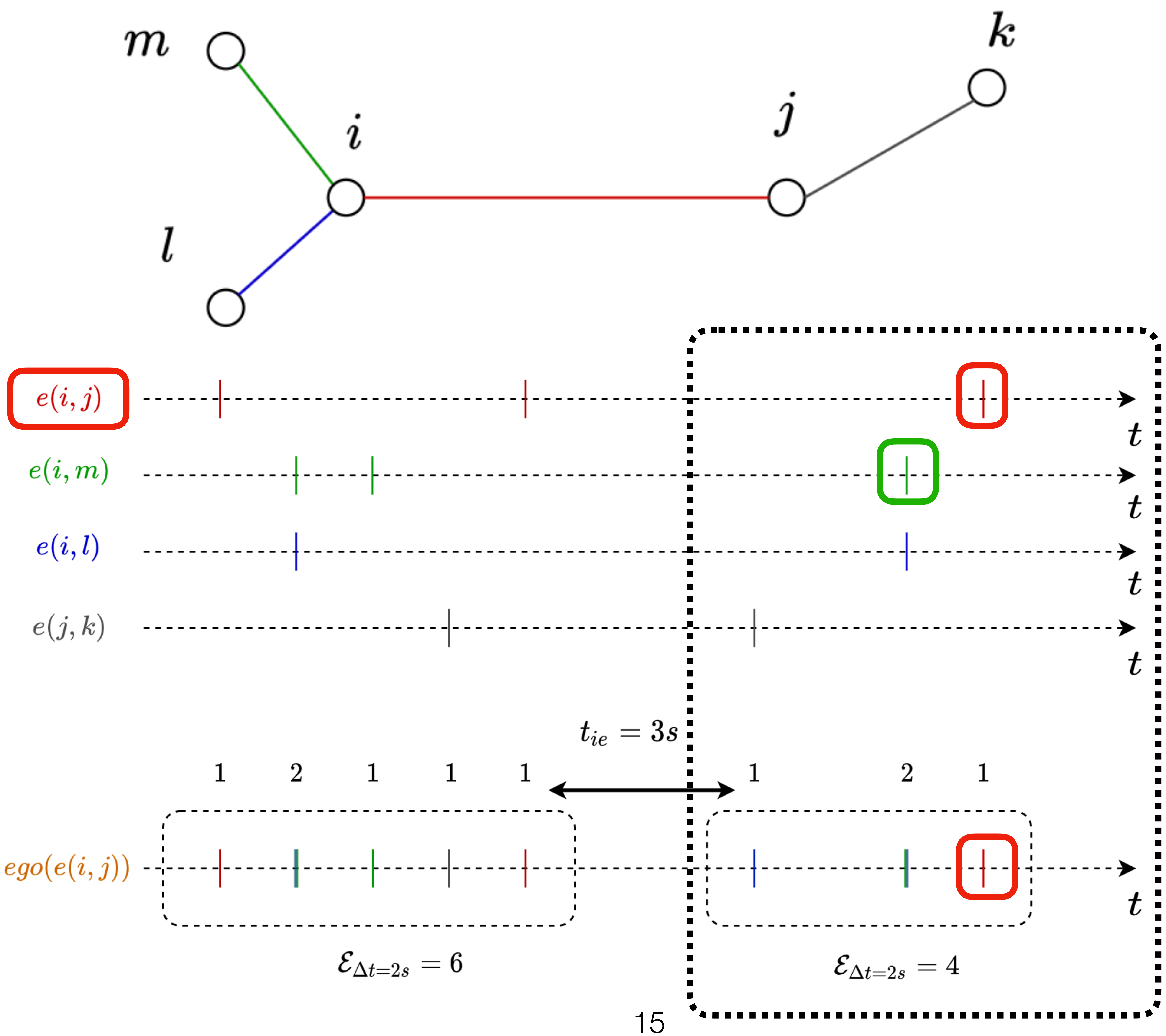

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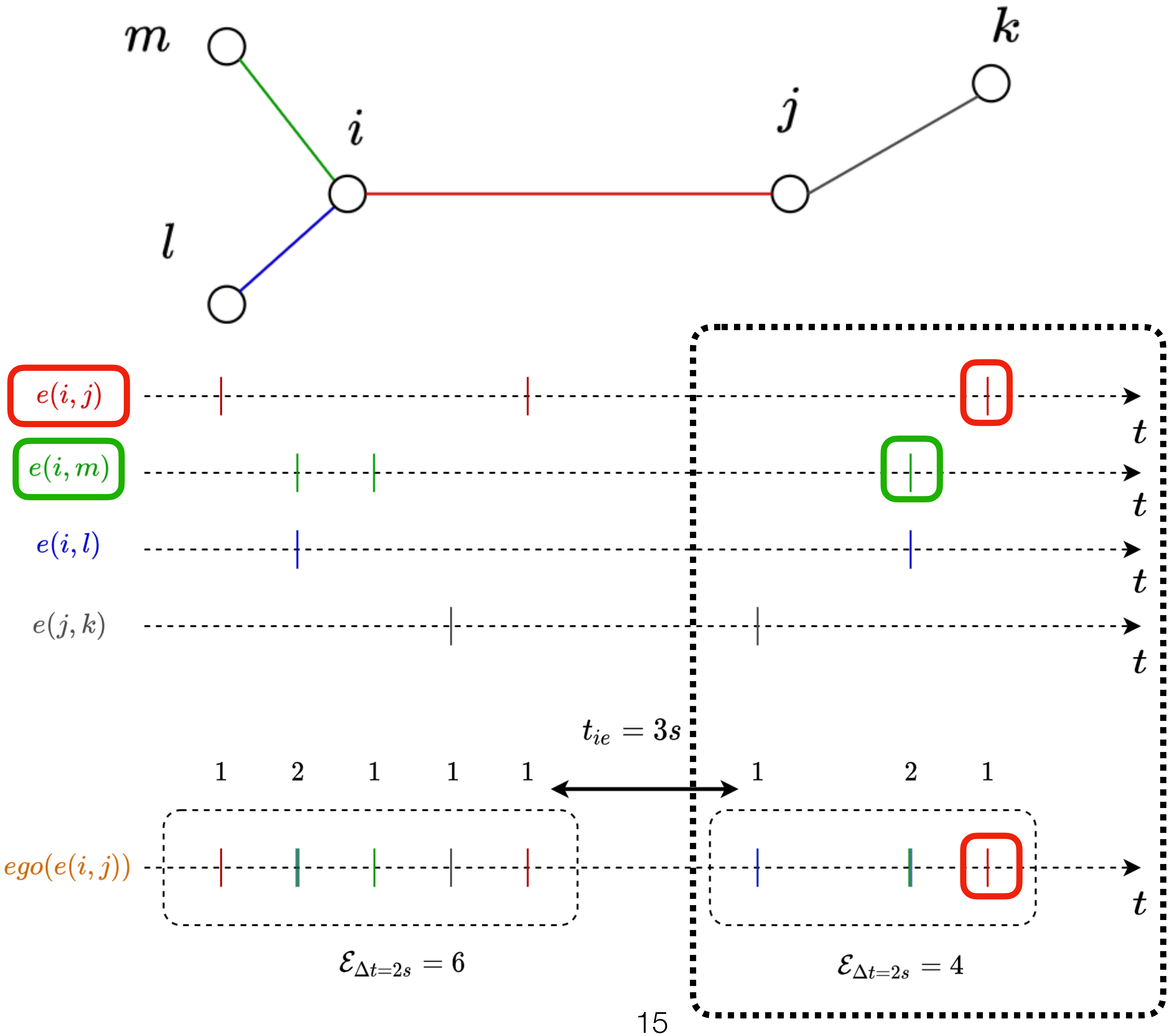

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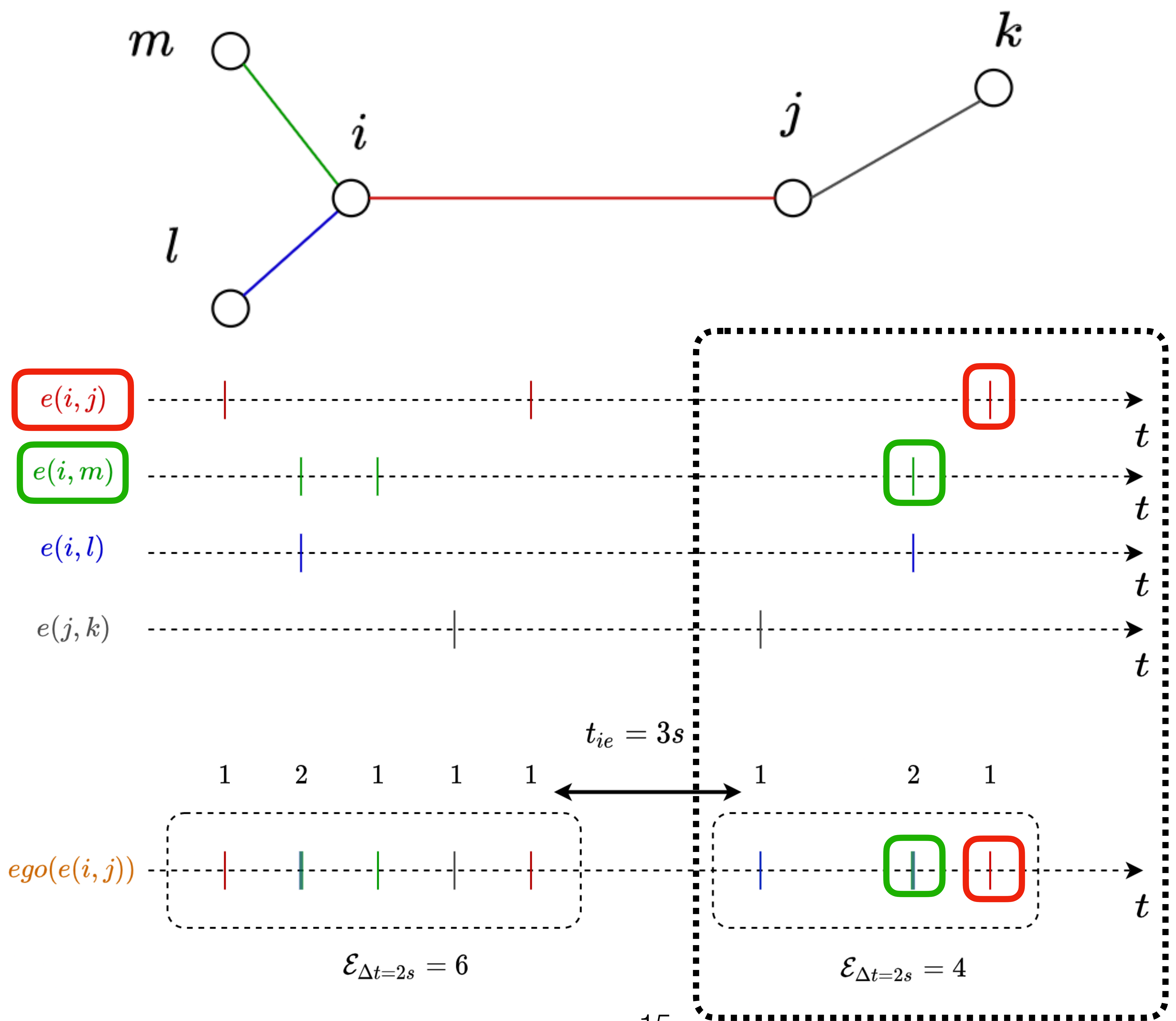

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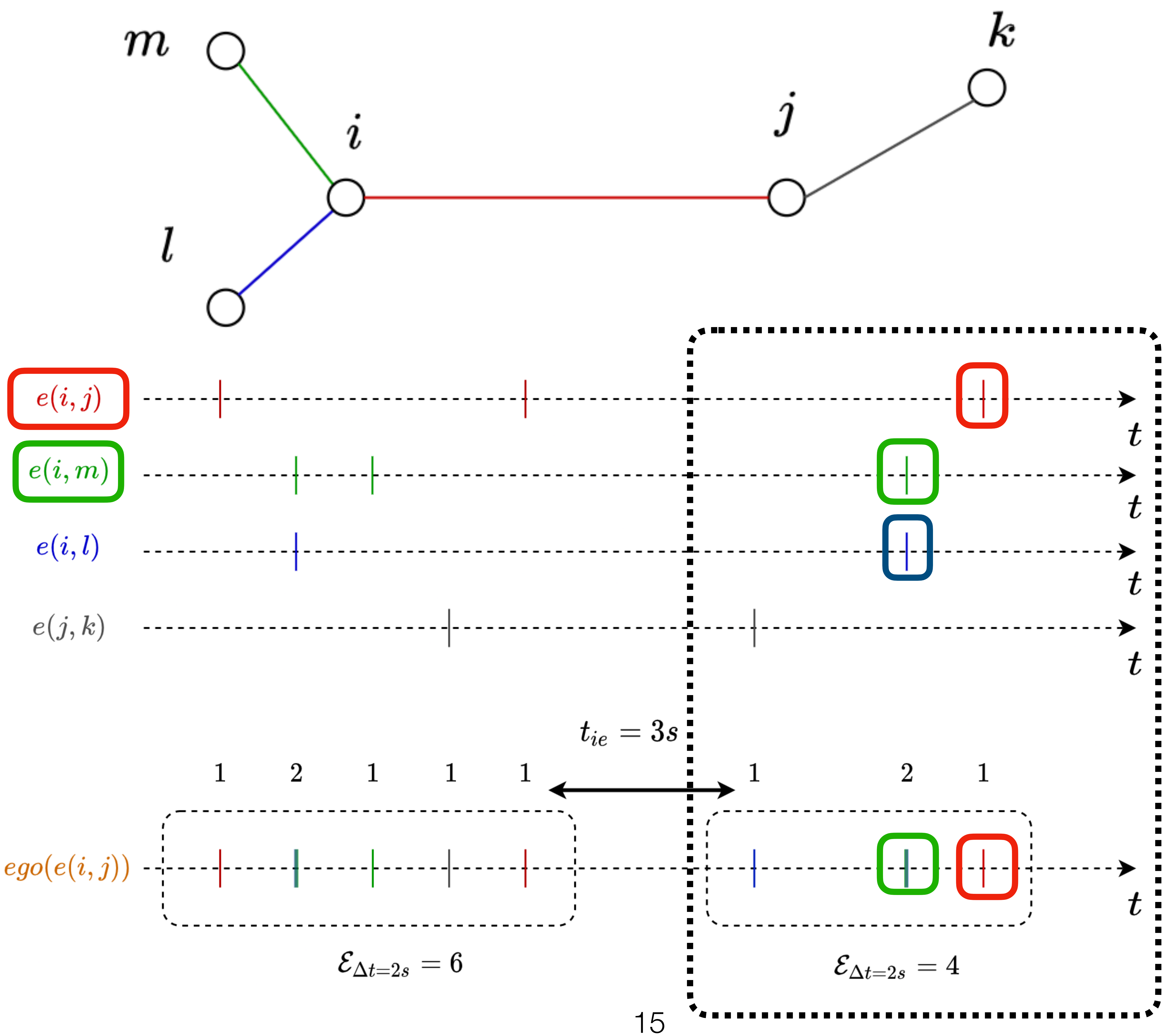

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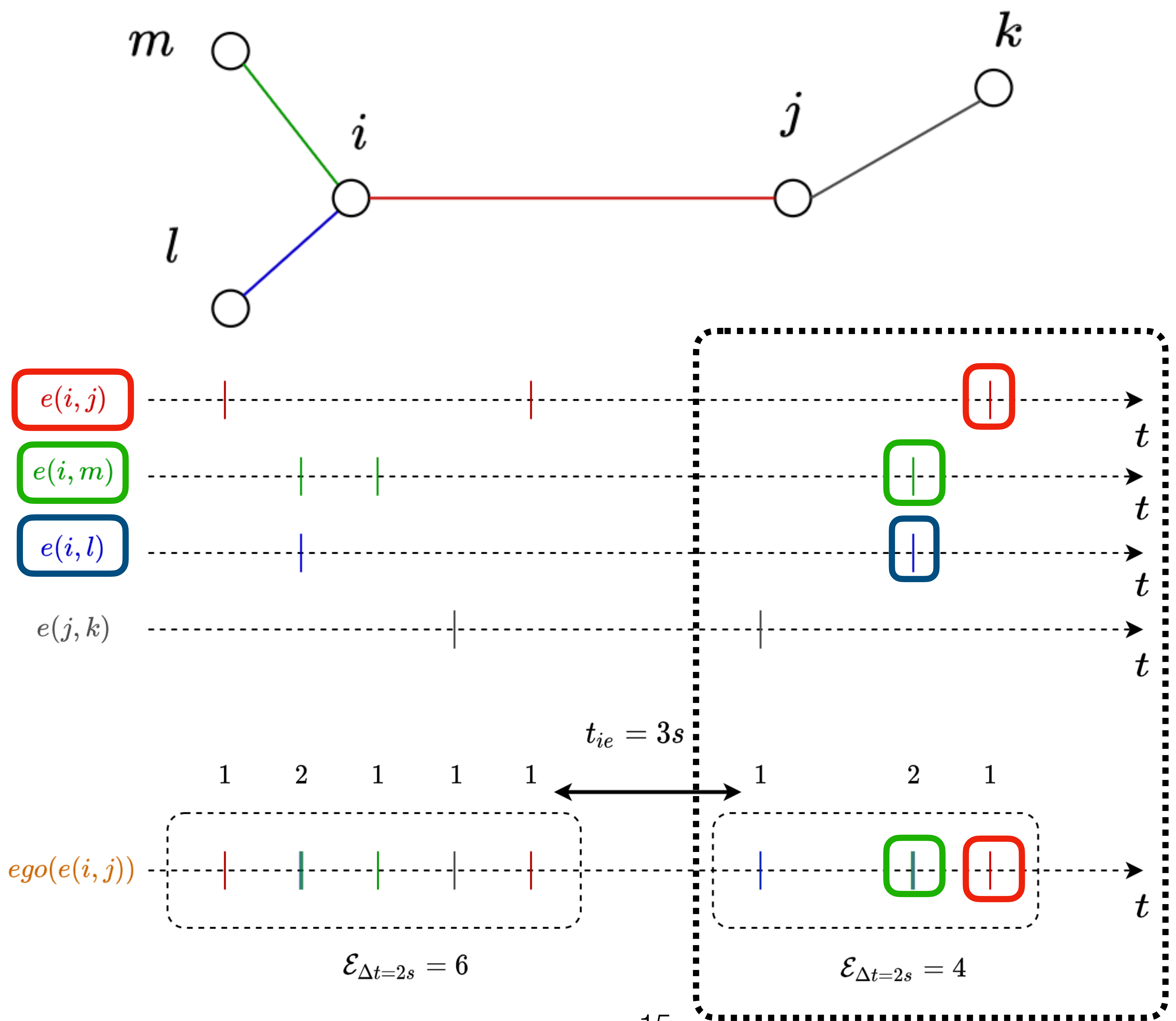

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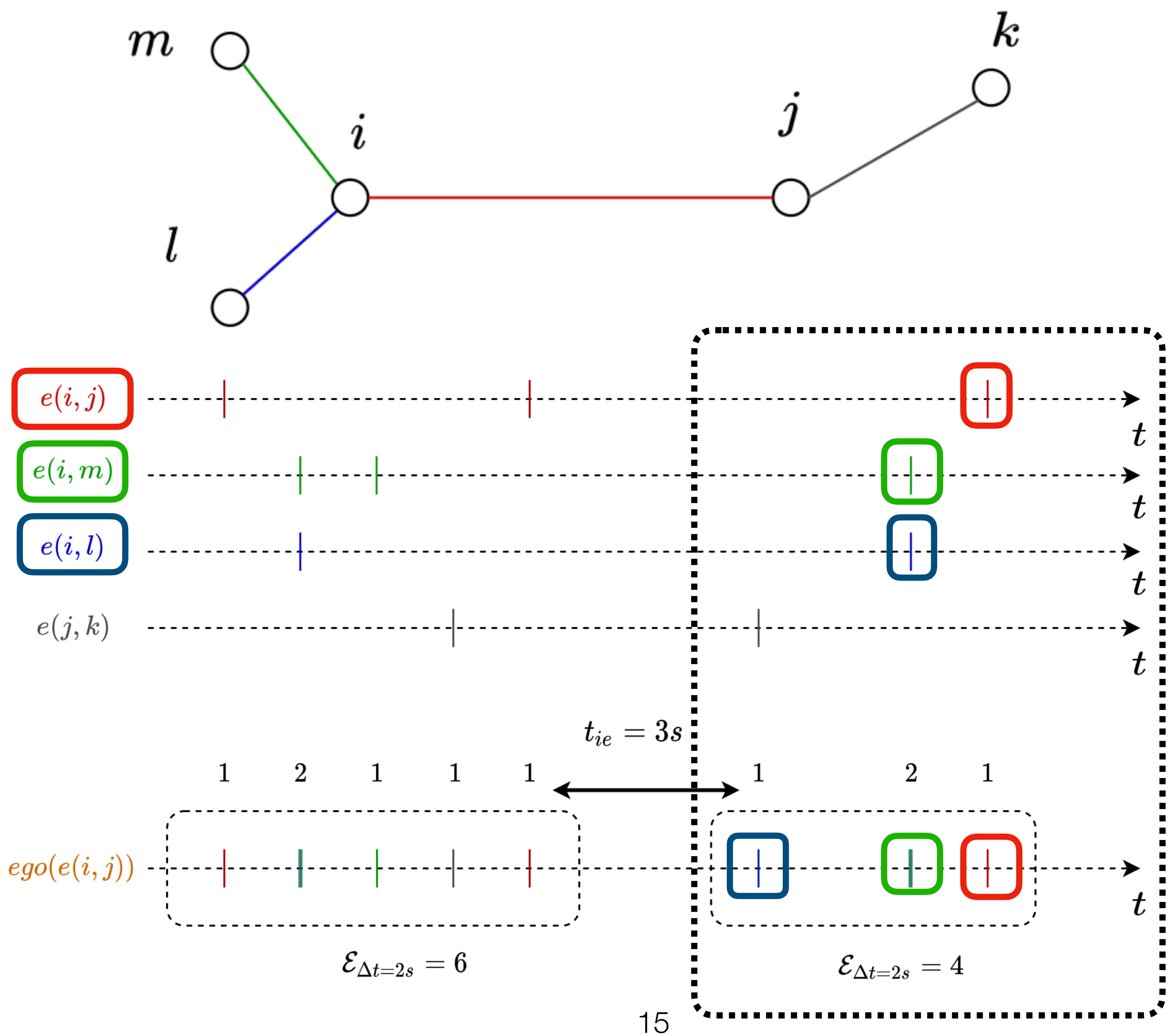

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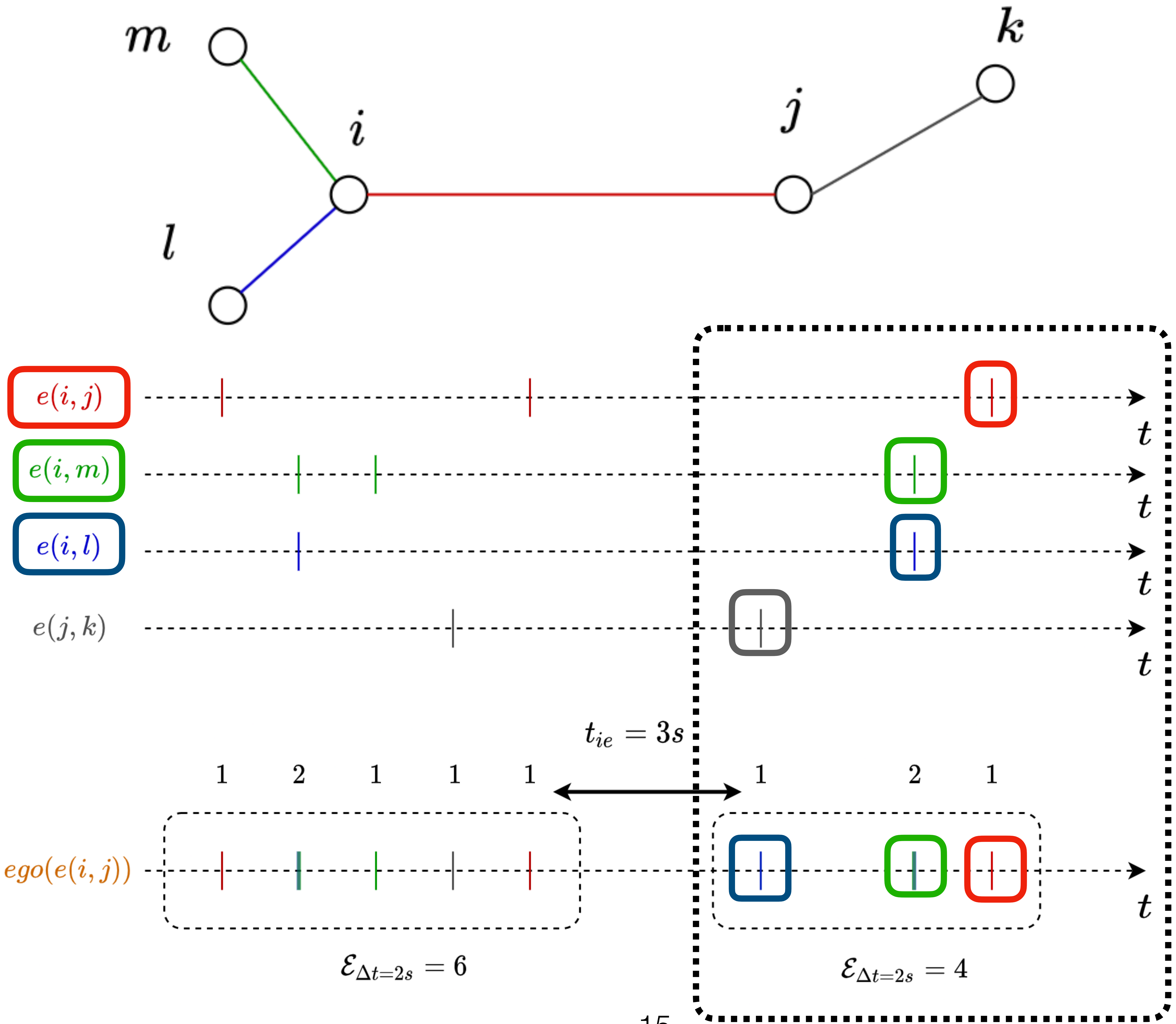

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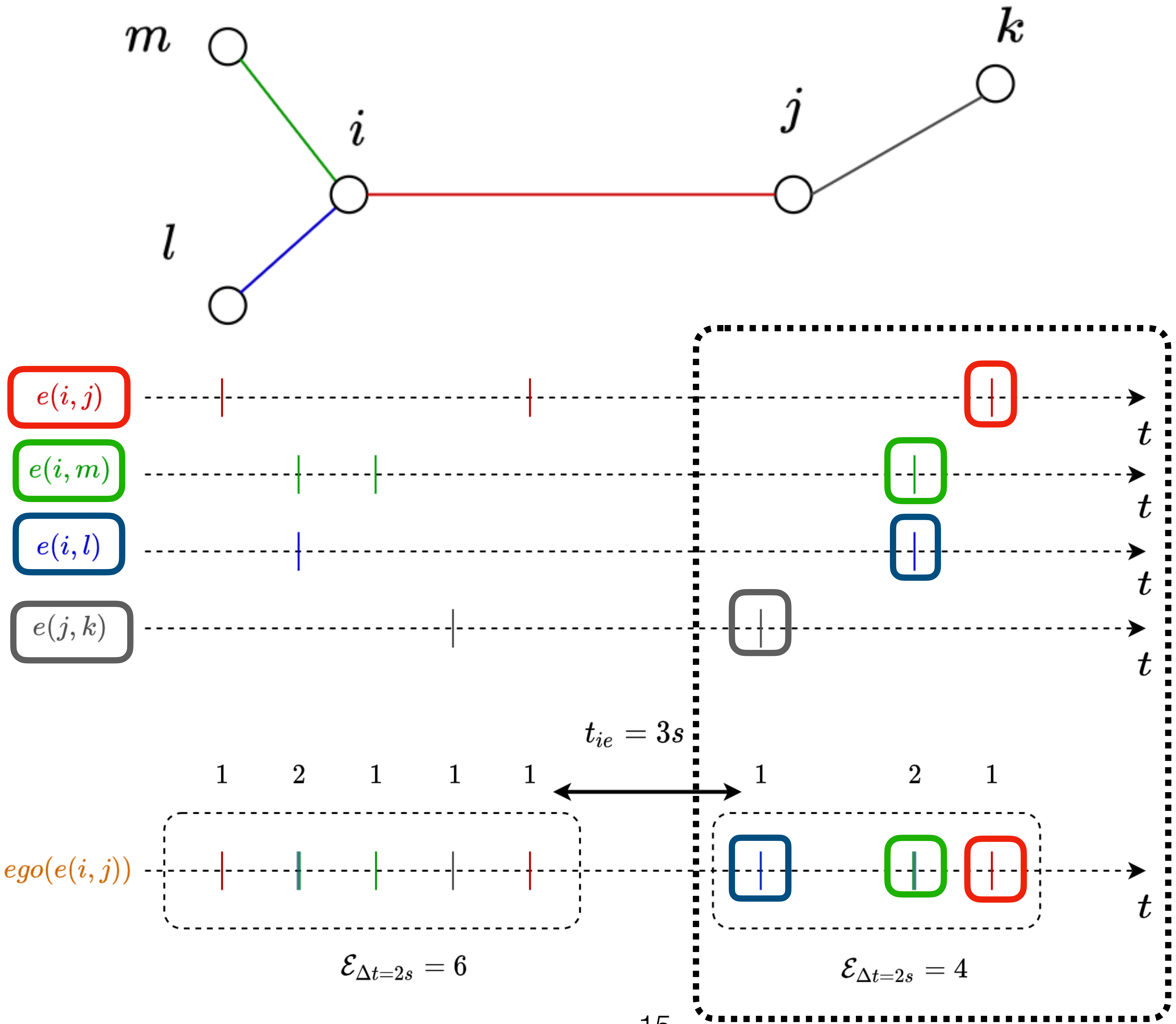

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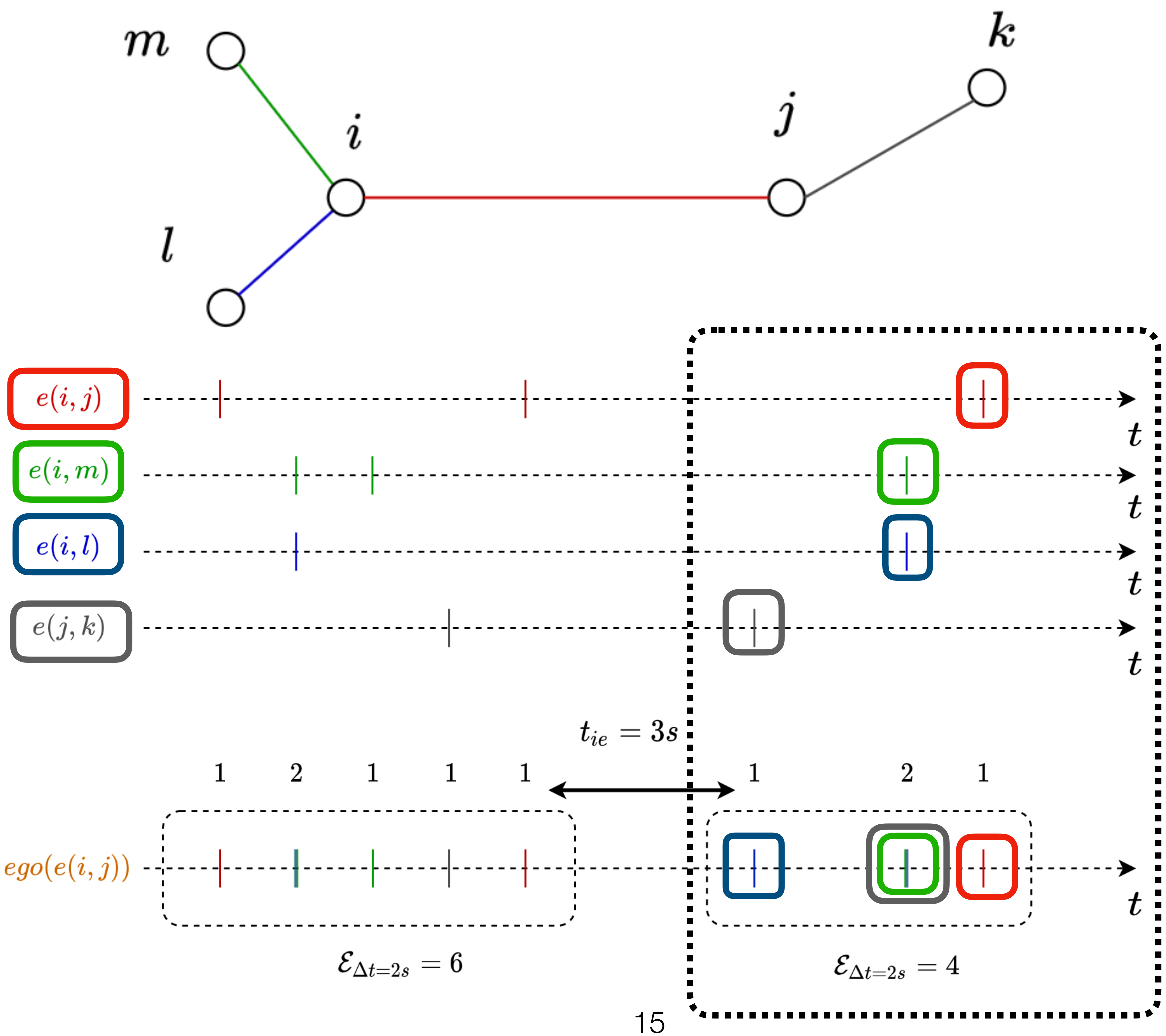
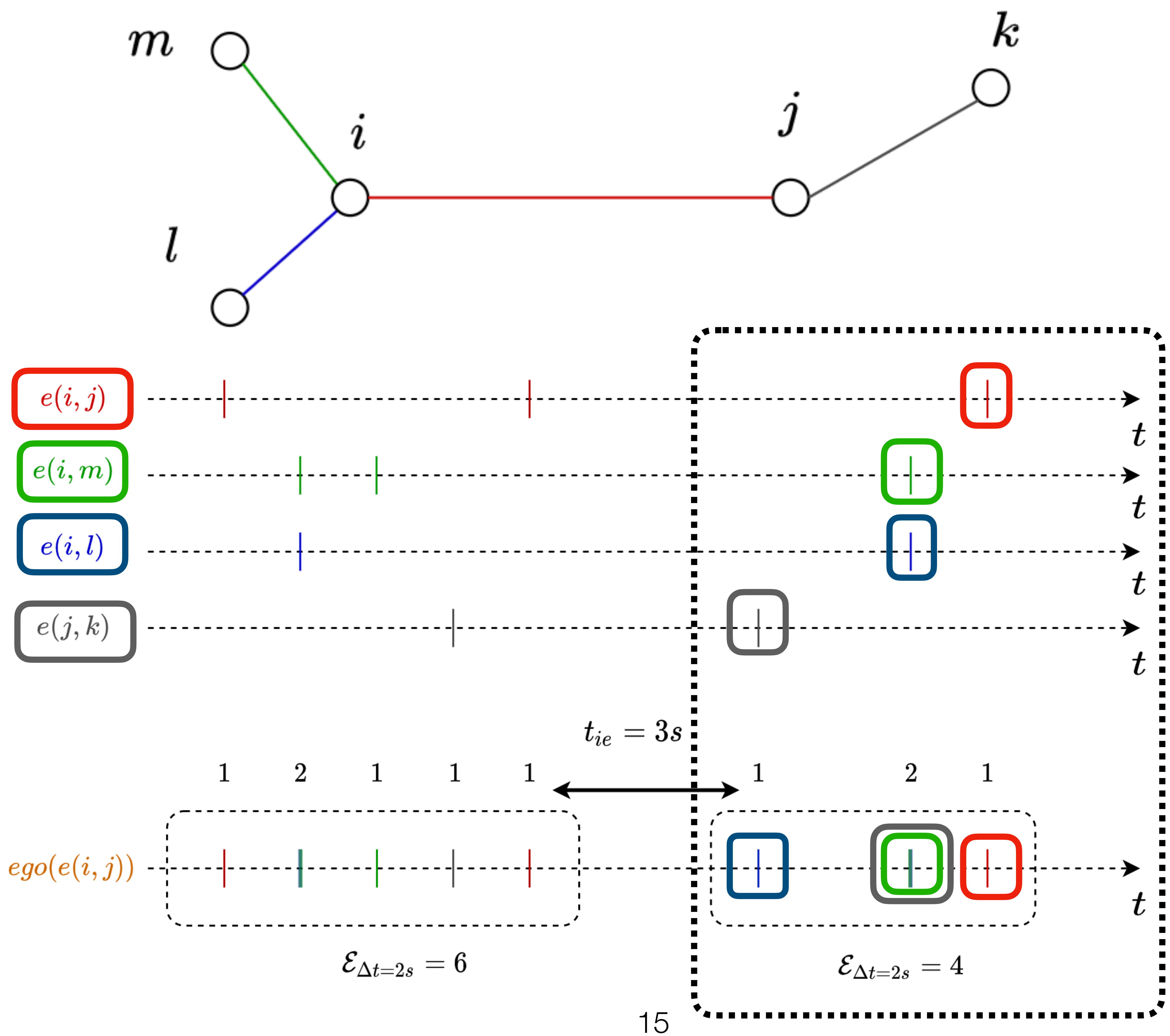
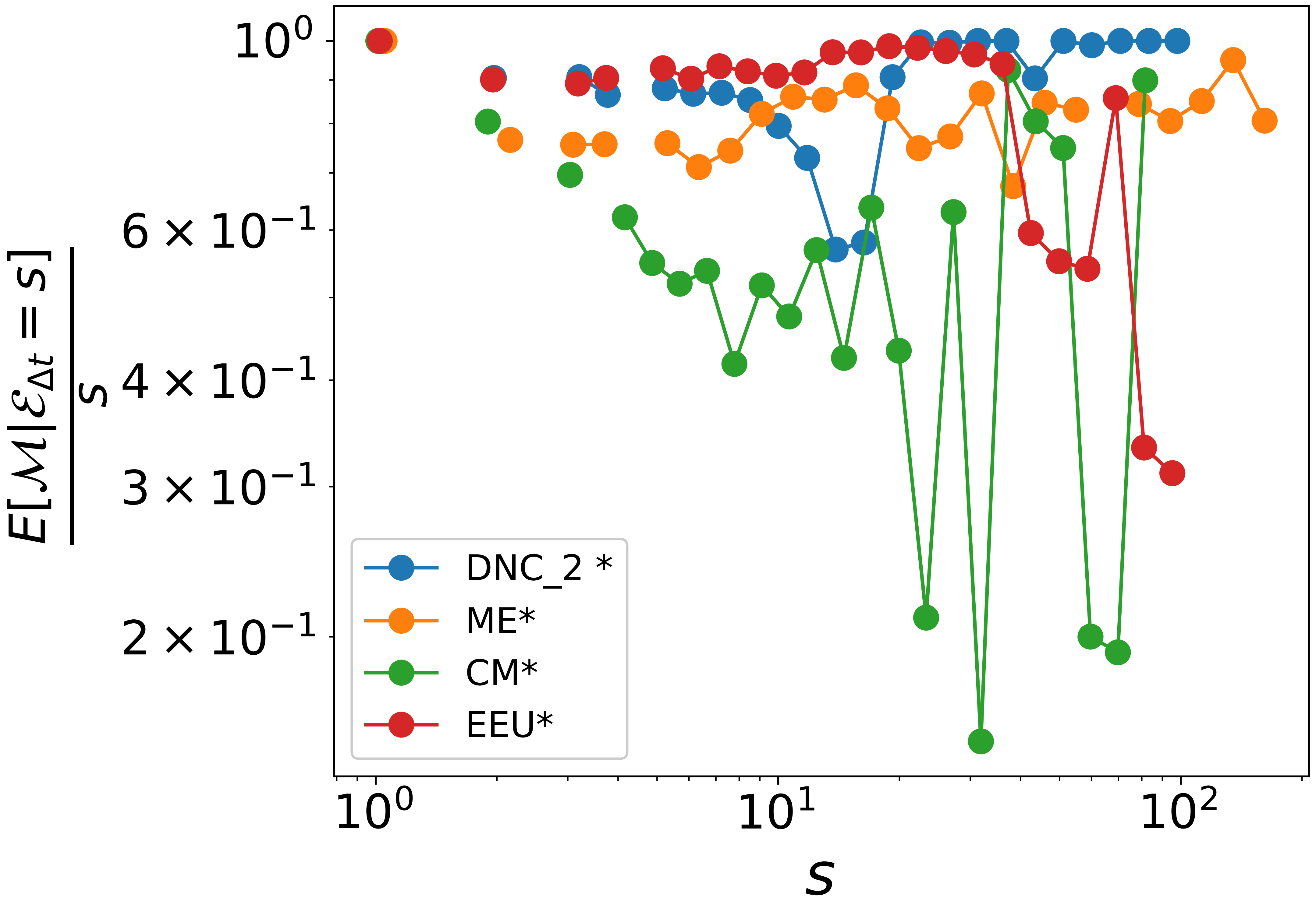


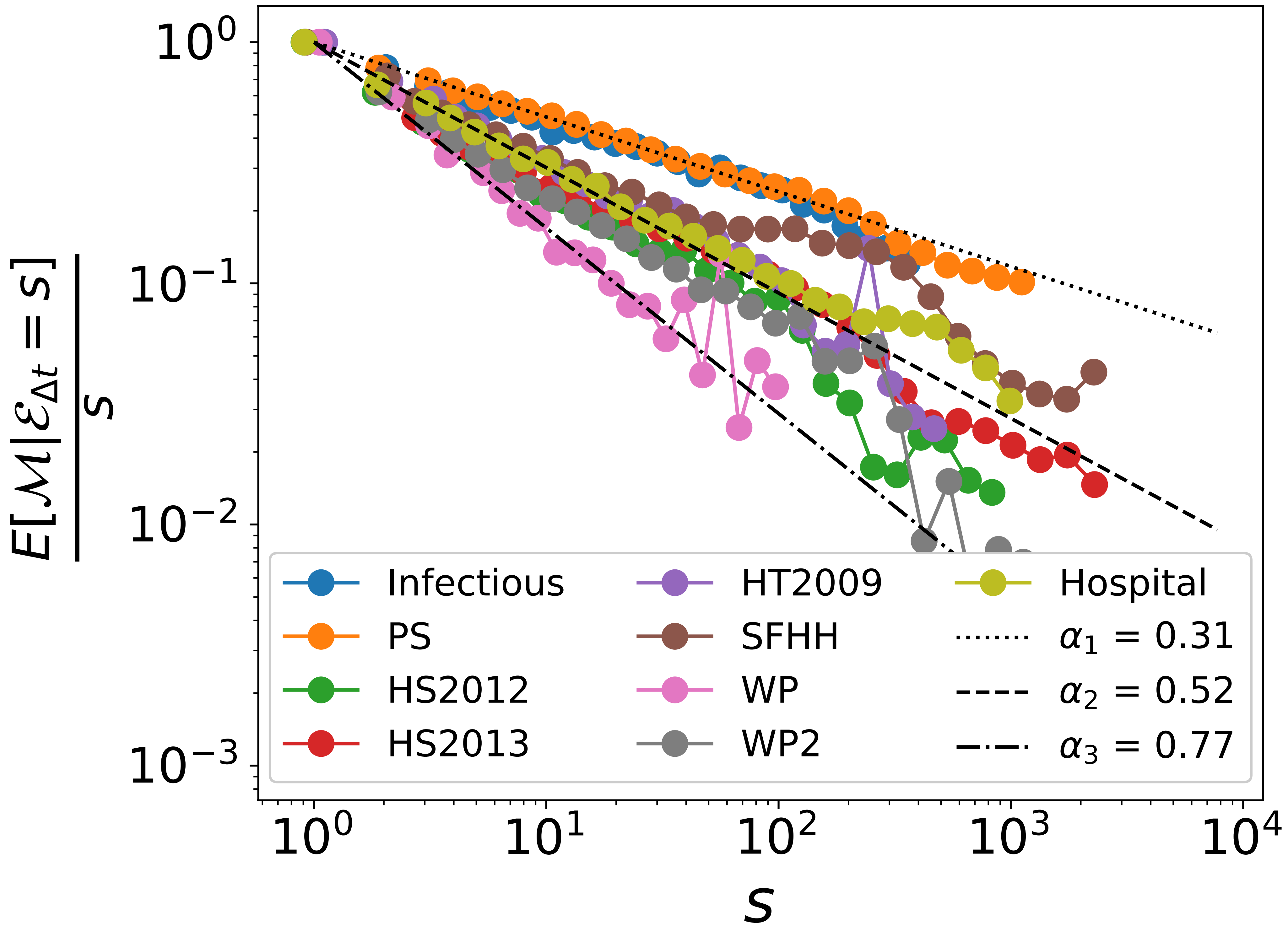
Number of active links during a train



$$\mathscr{M} = 4$$

How many links are active during each train?





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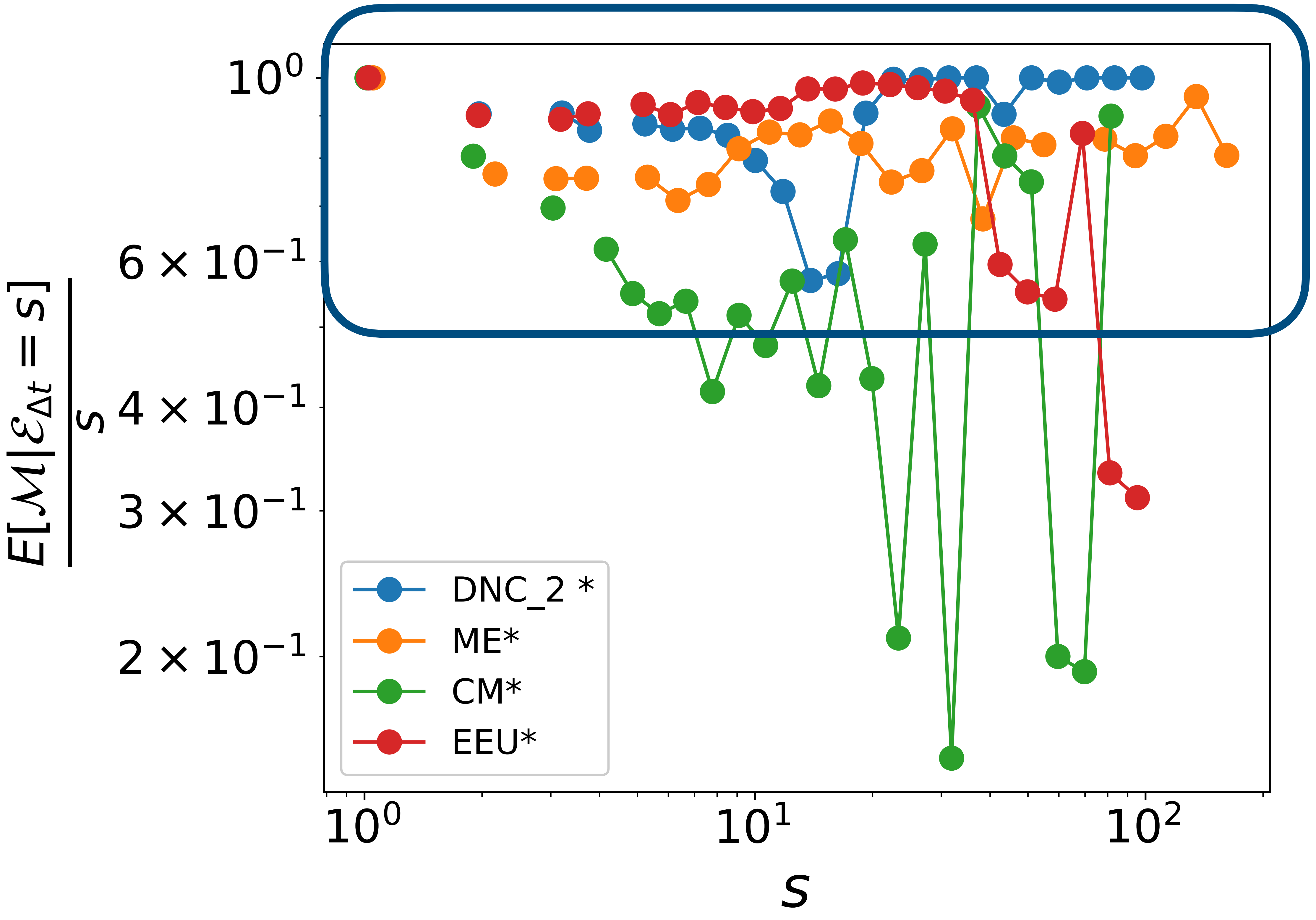

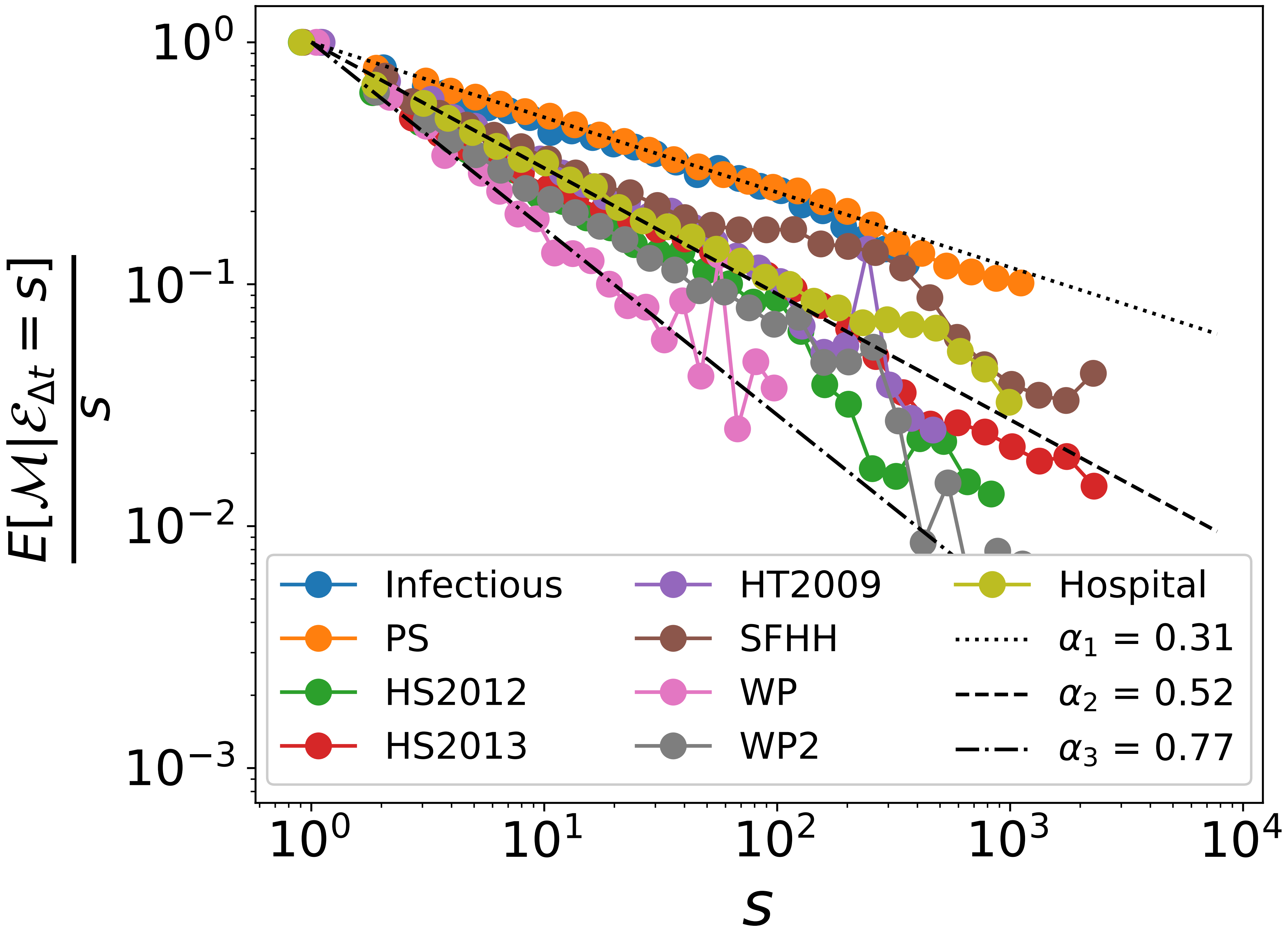

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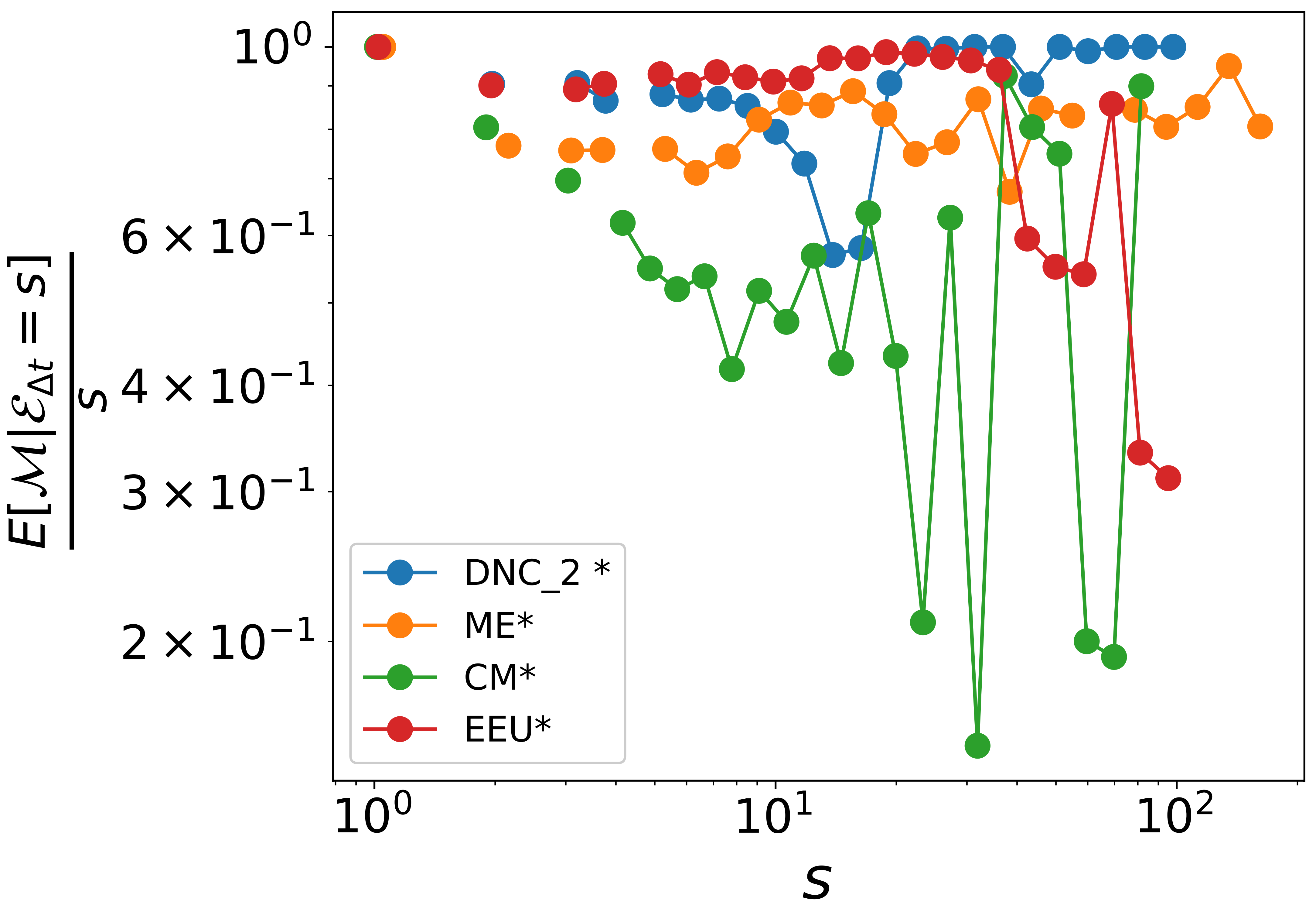

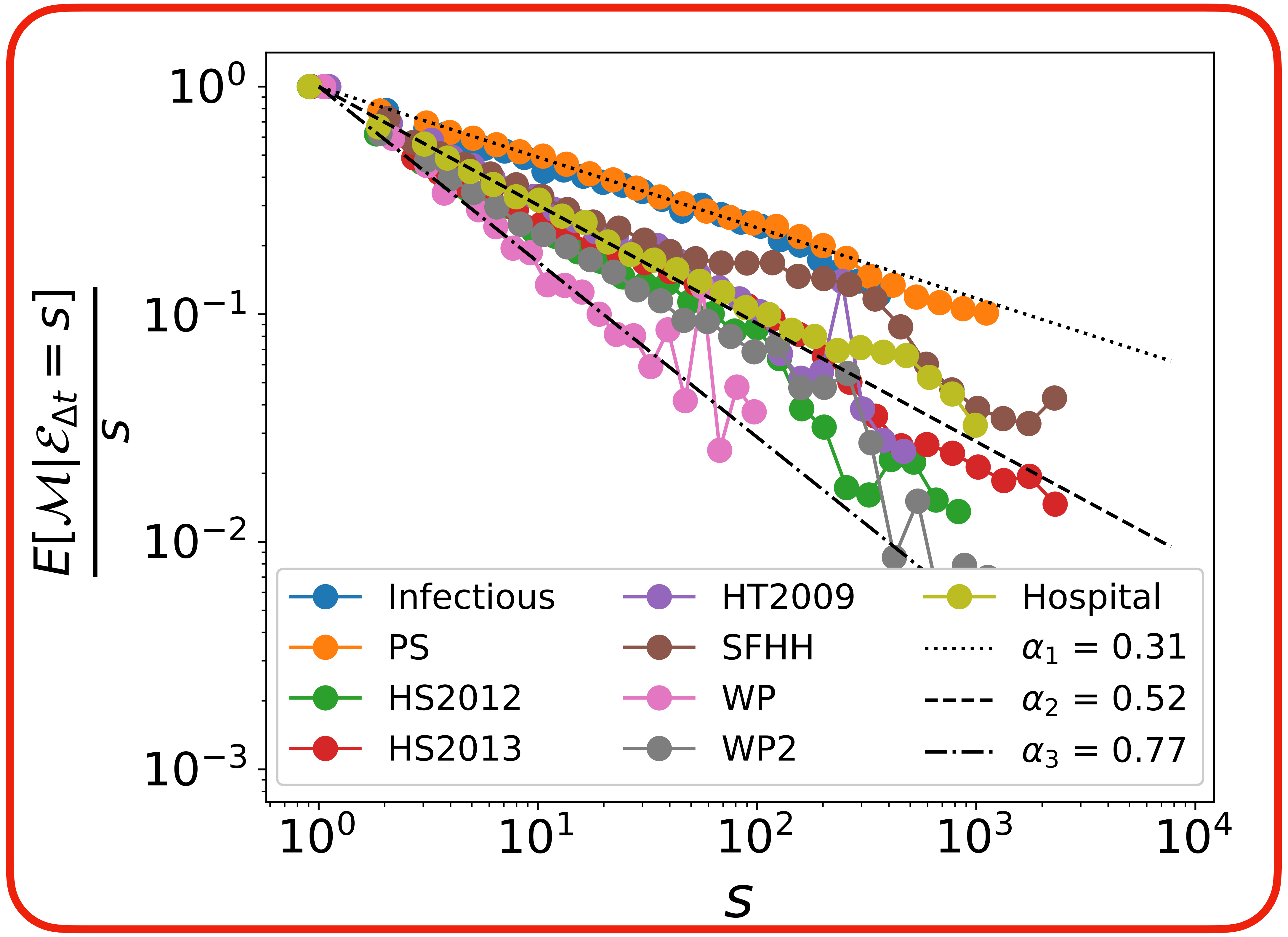

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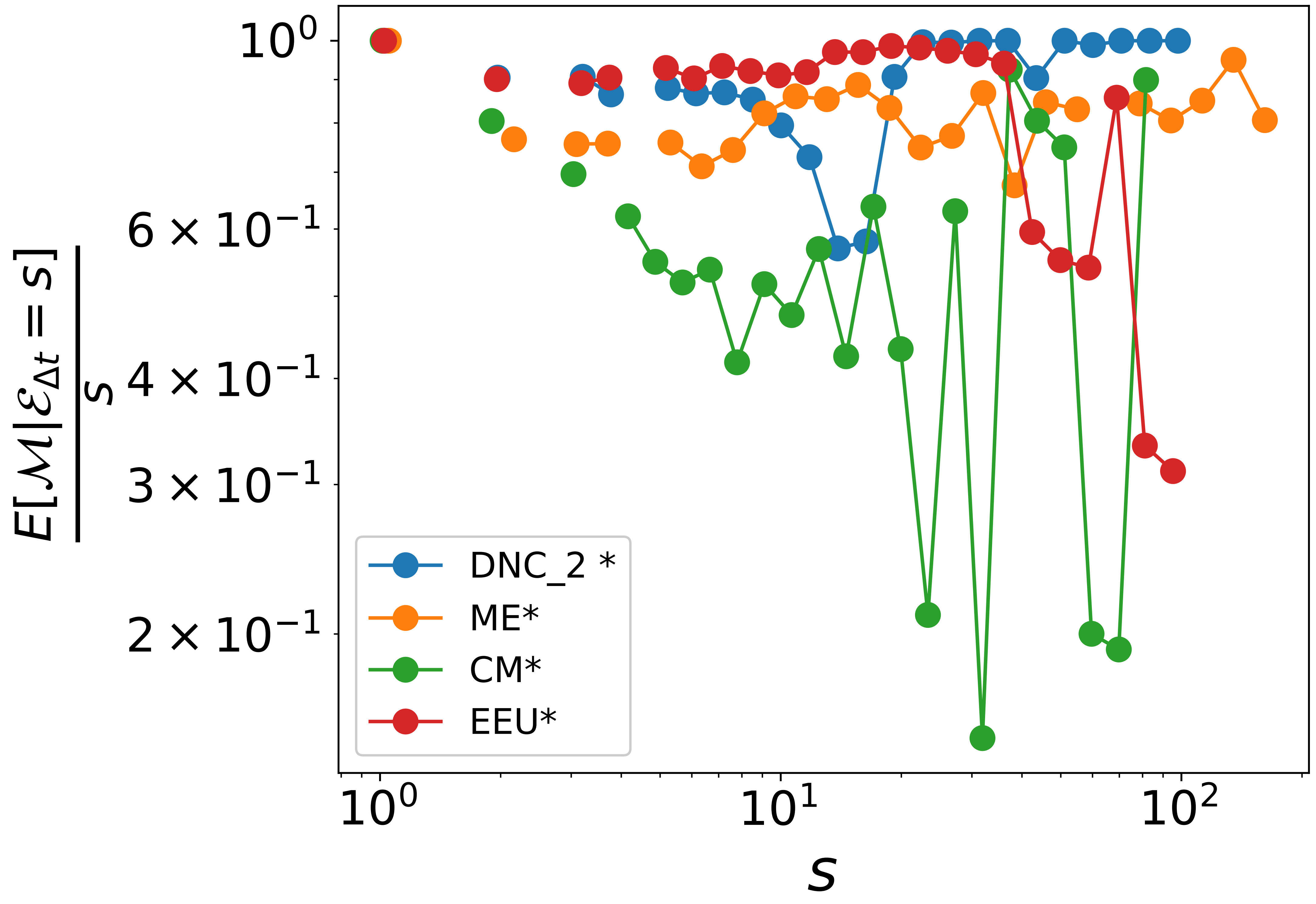
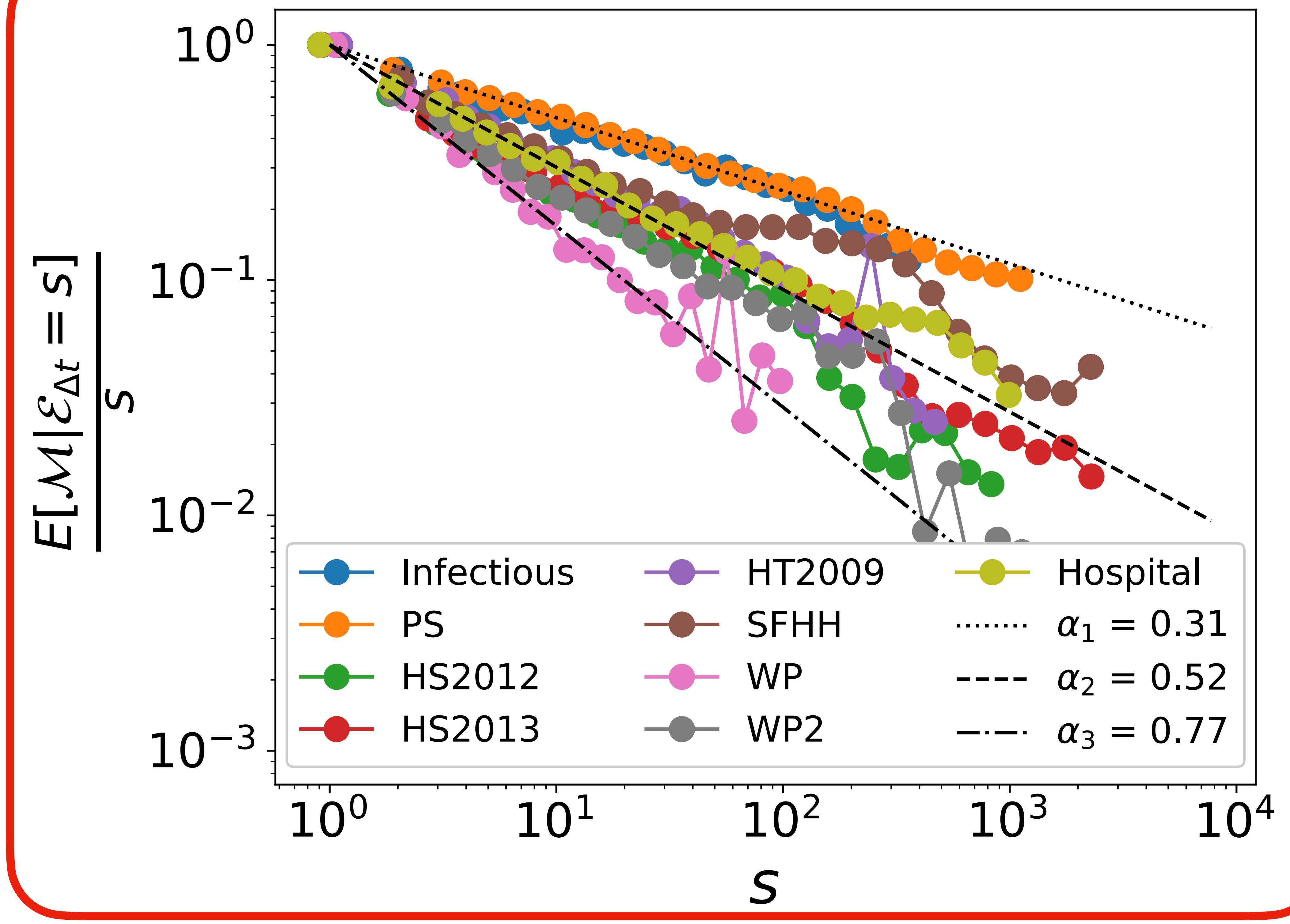
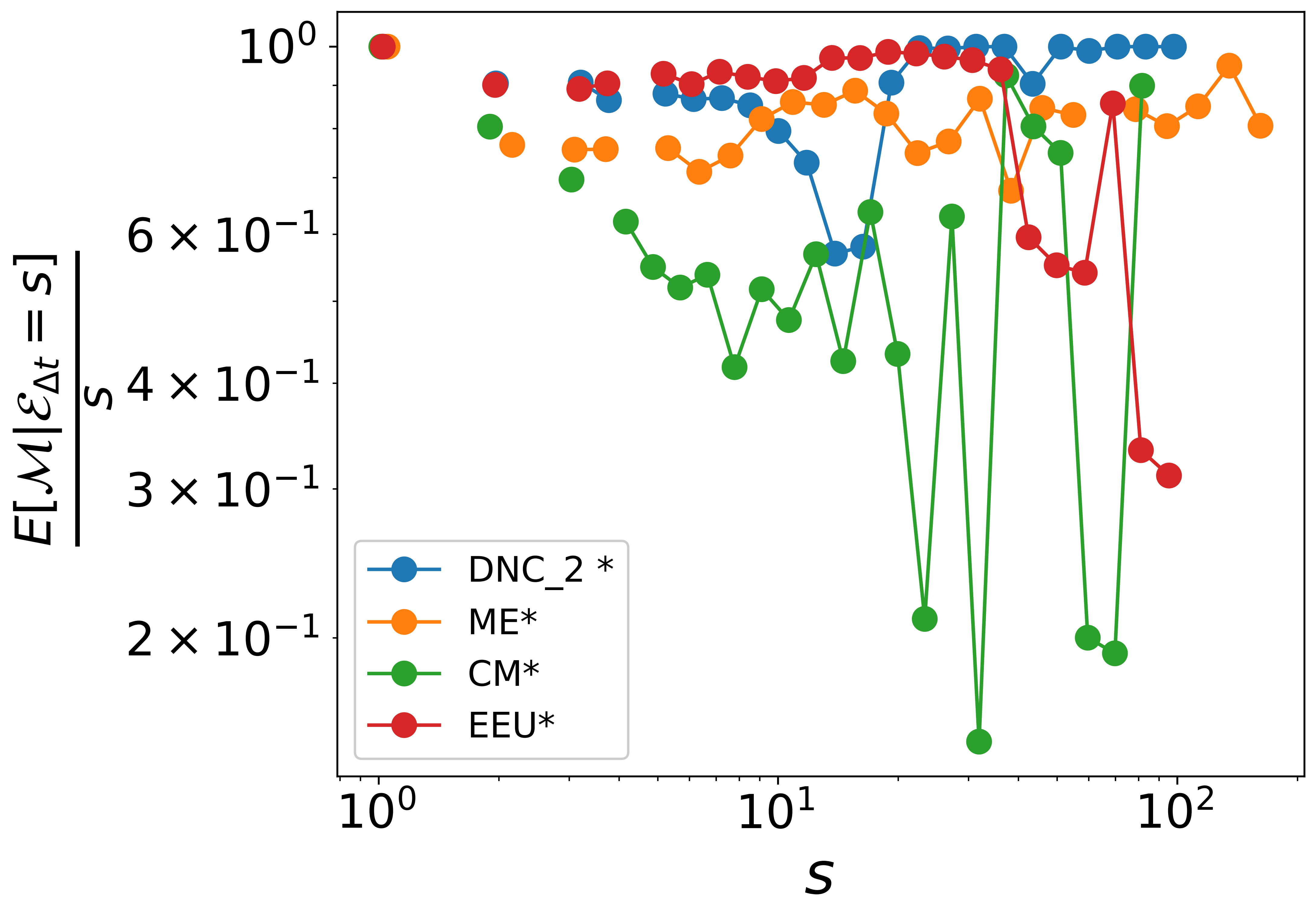
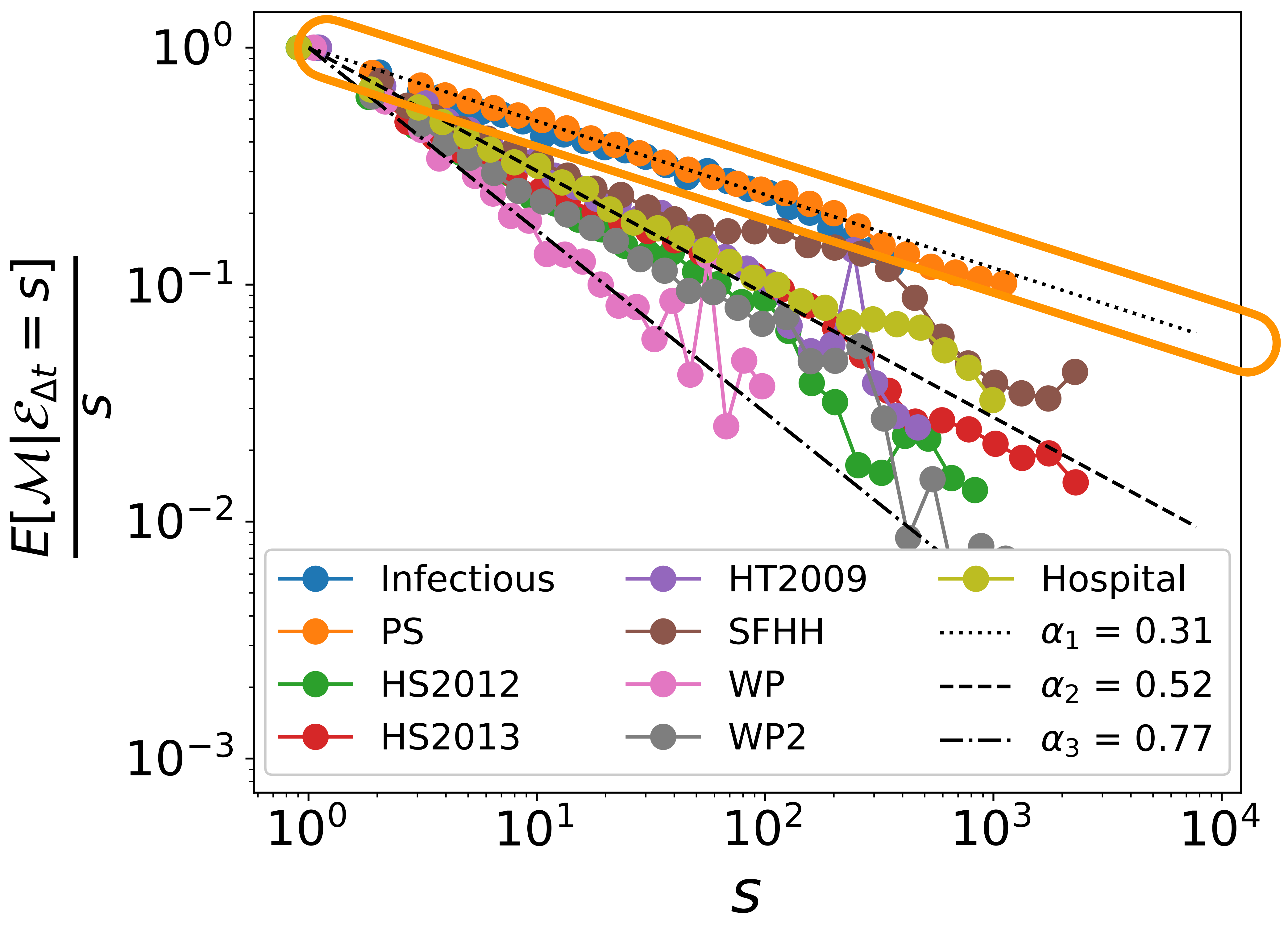




In physical contacts, the number of active links during a train is influenced by the social context!

How many links are active during each train?





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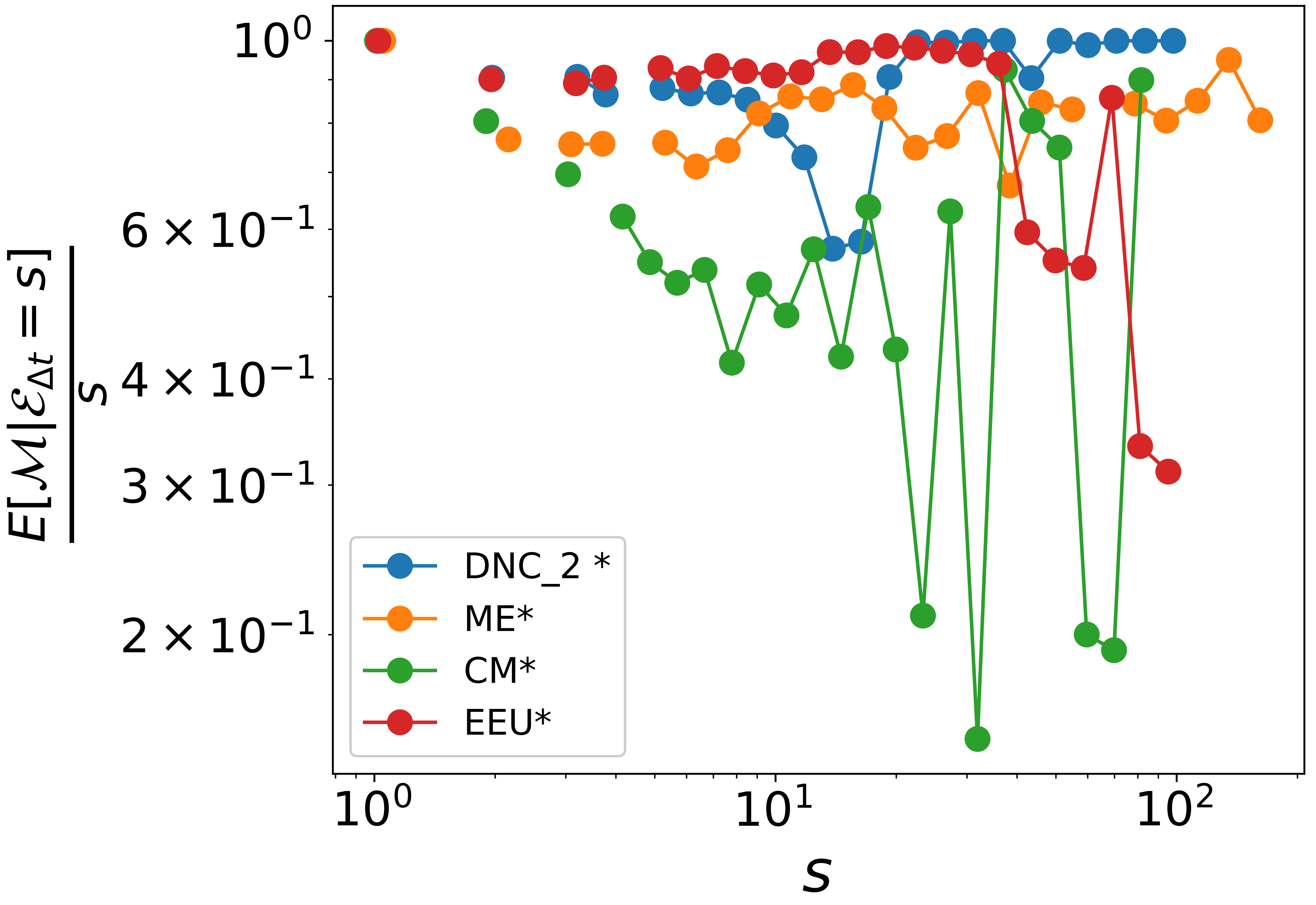

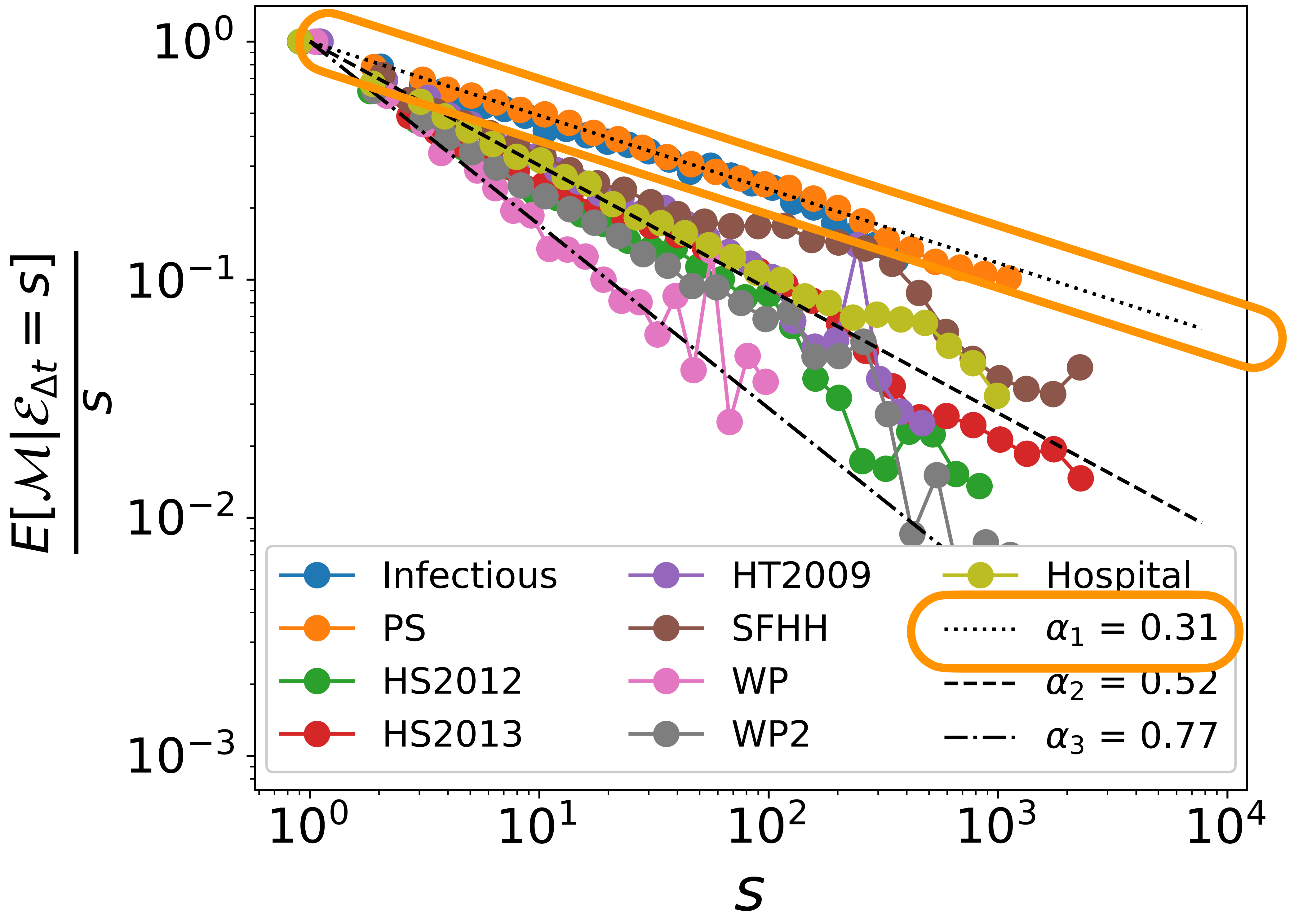

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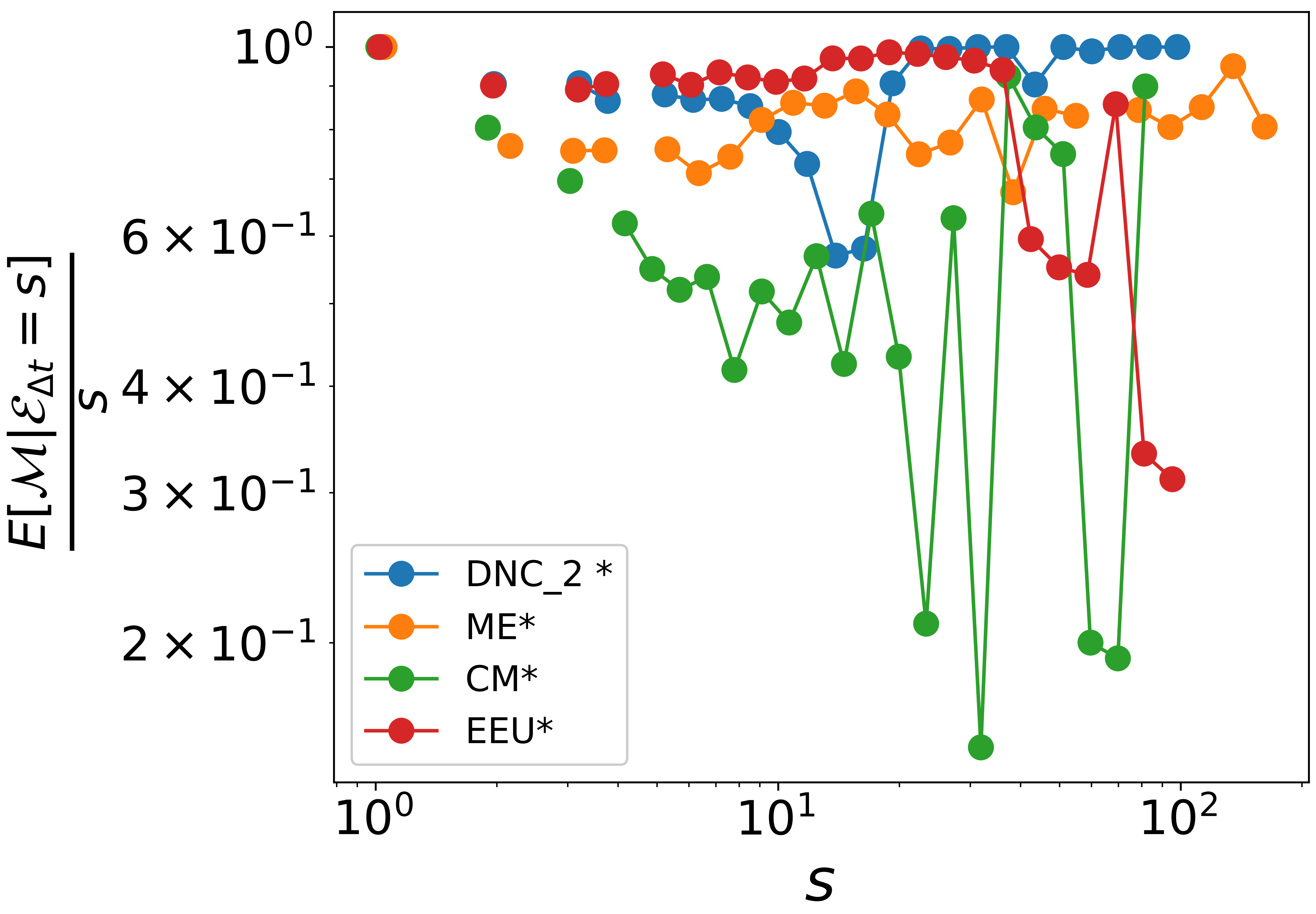

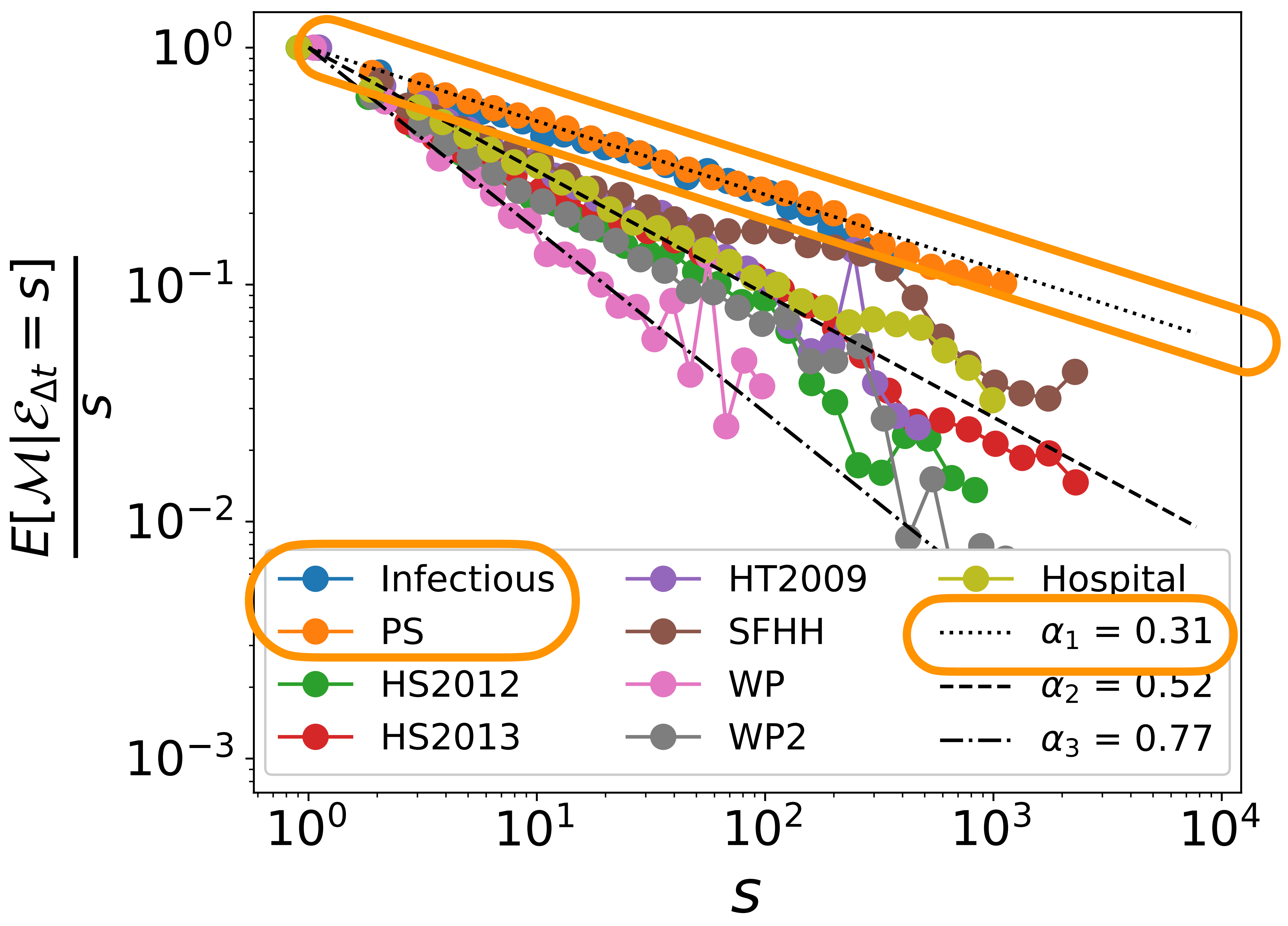

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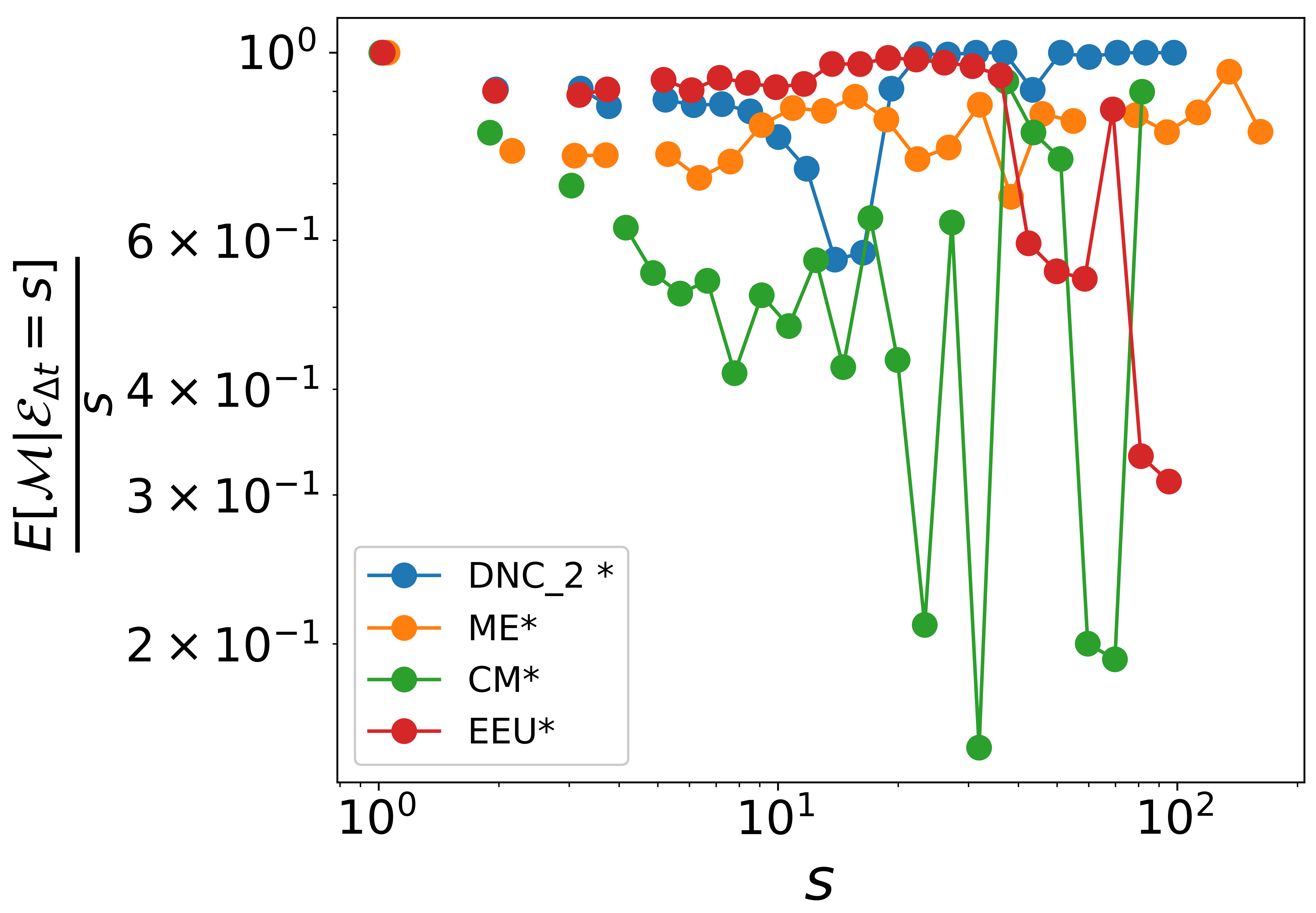

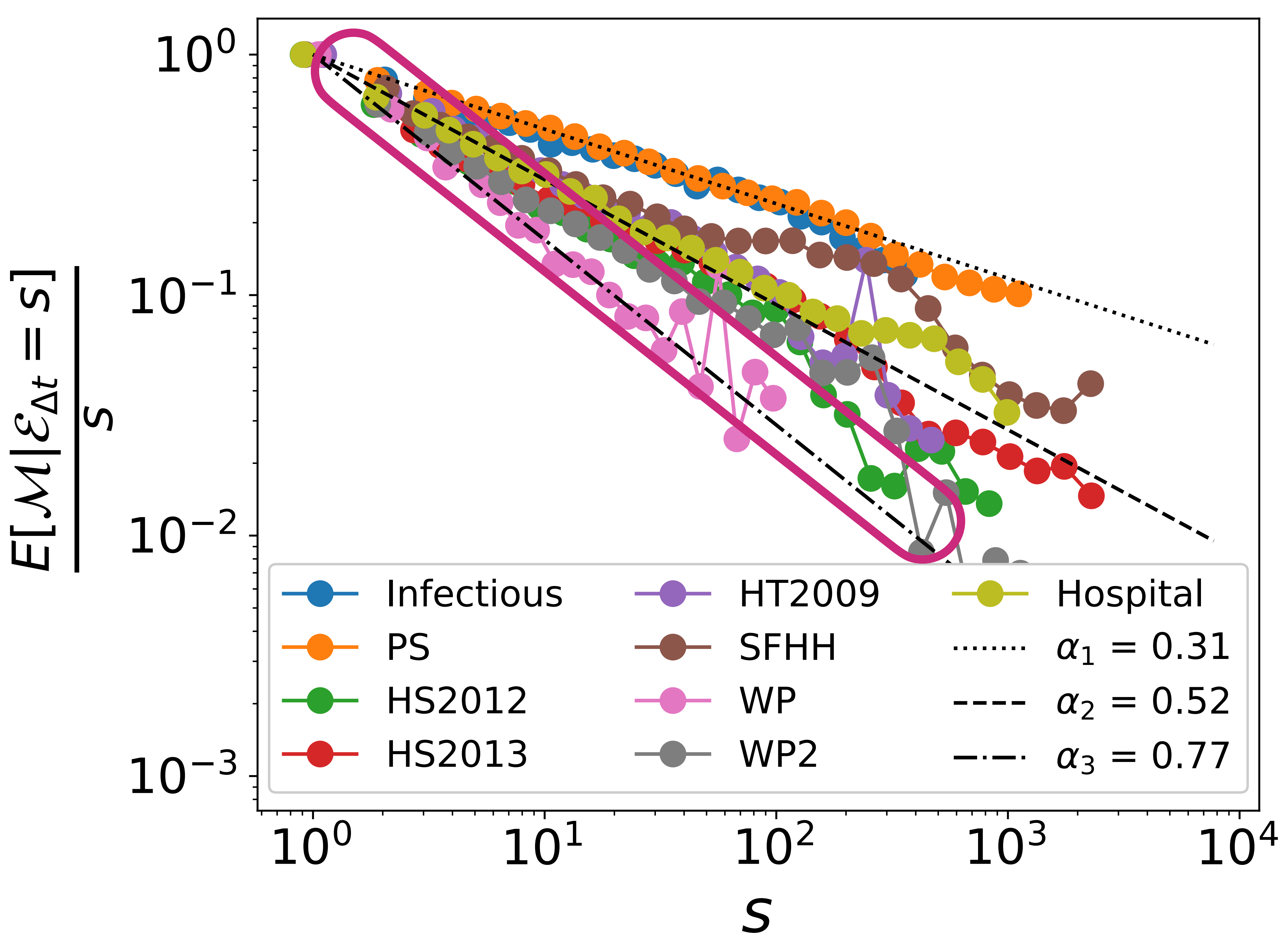

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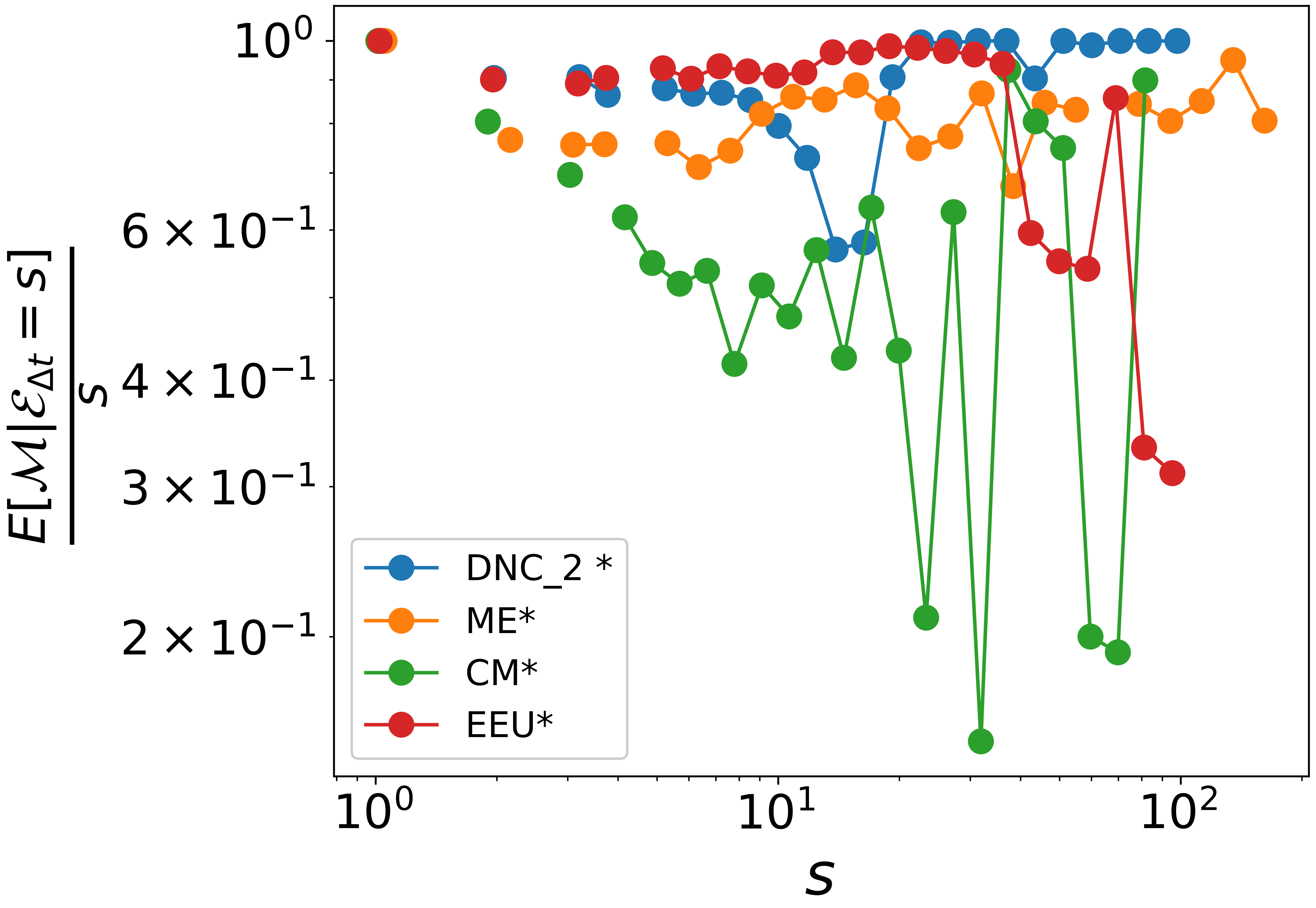

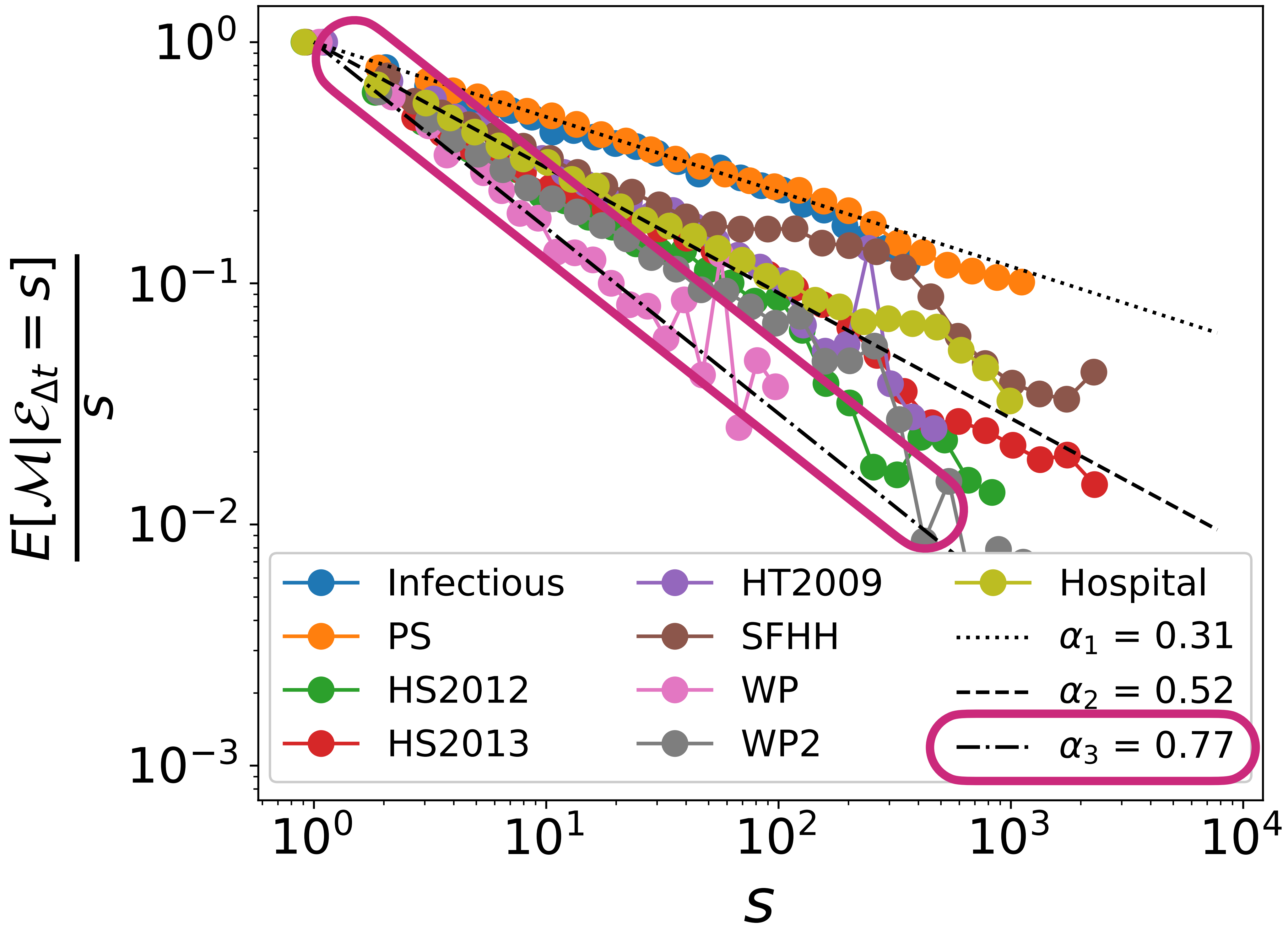

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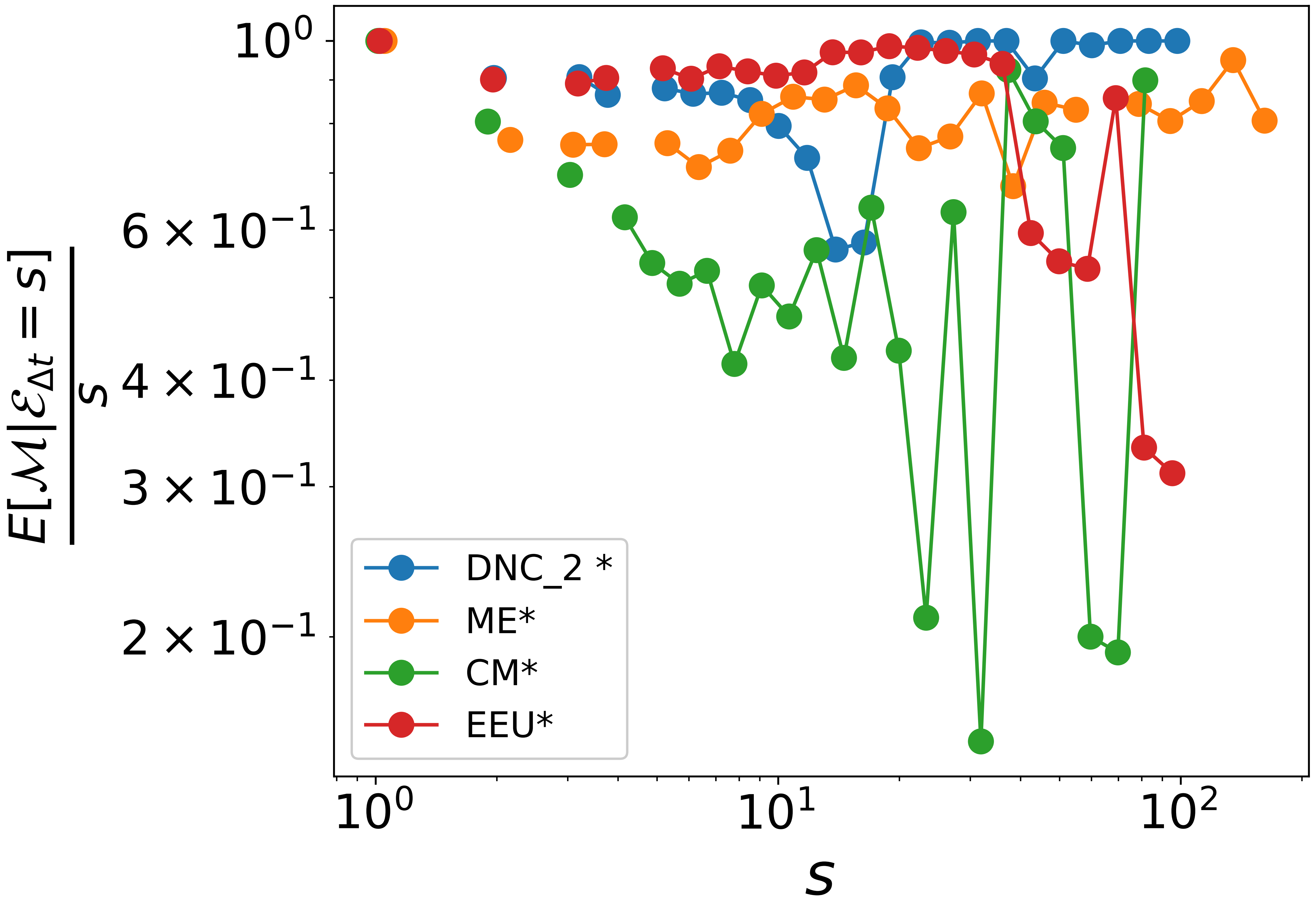
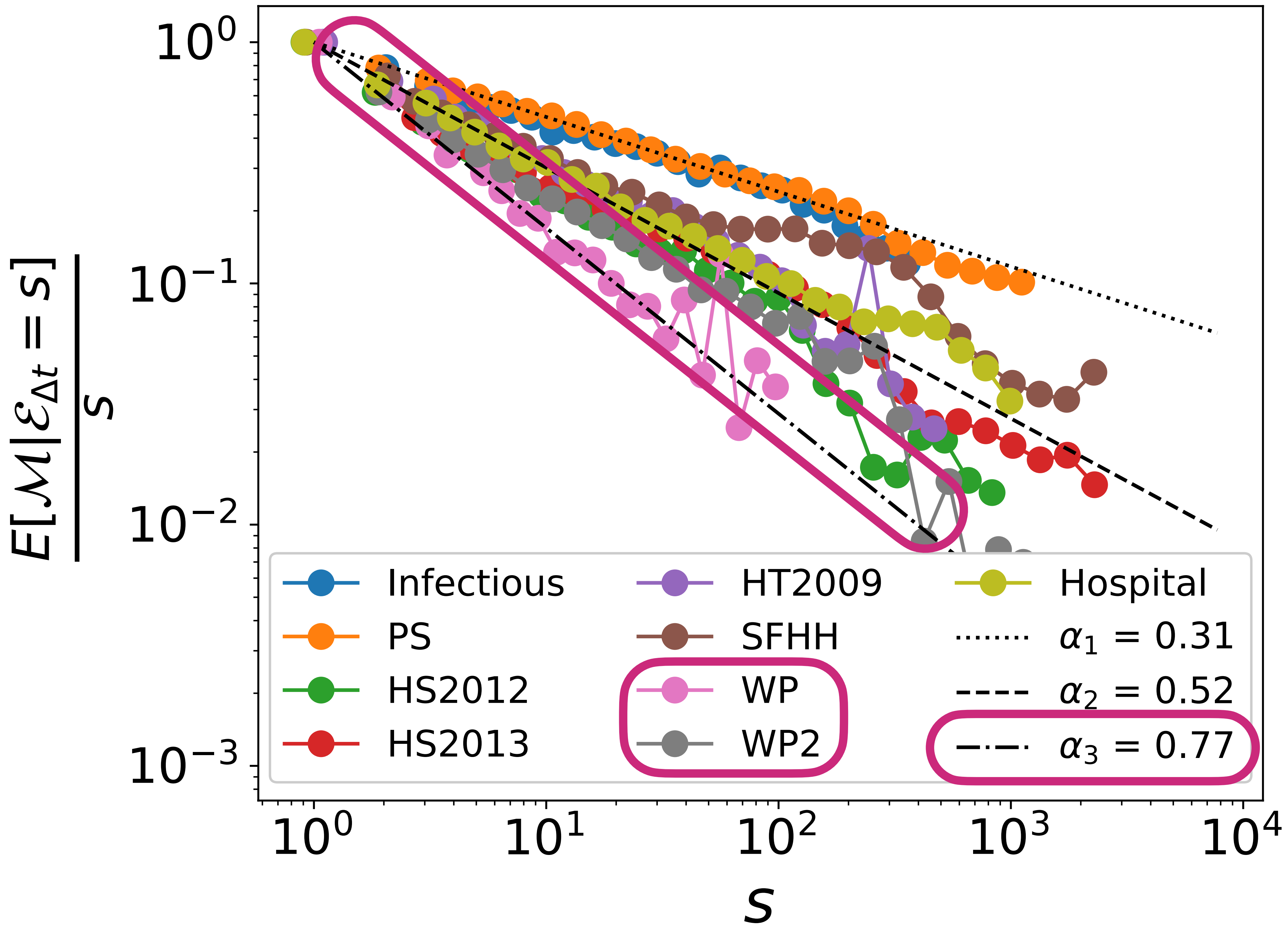
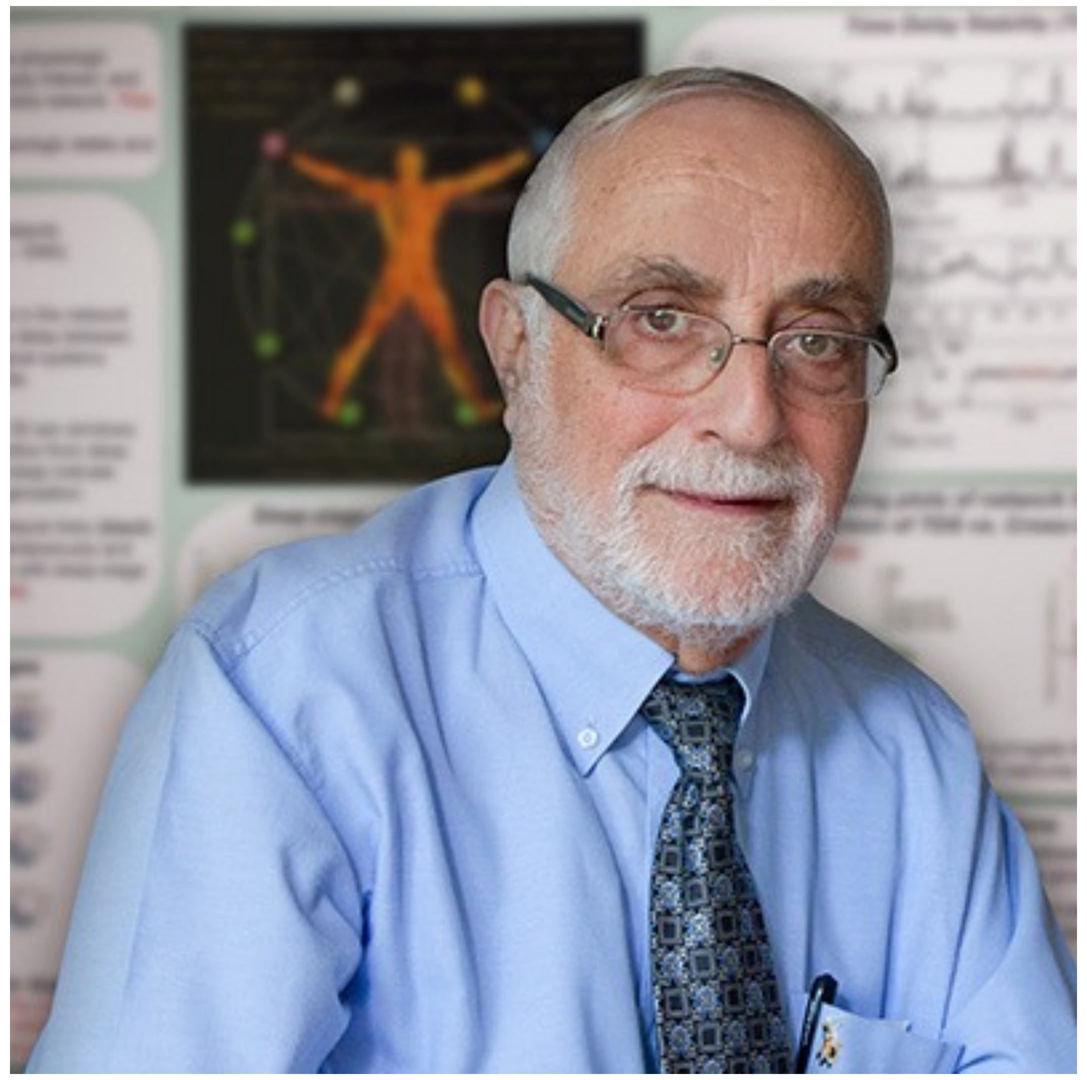




Take home messages

- Contacts close in time tend to happen also close in topology
- Temporal correlation among neighbouring links is particularly evident in virtual contacts and Infectious datasets
- The number of active links during a train reflects different social contexts

Thank you!



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Dank je wel!

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Topological–temporal properties of evolving networks

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